# Multi group cosmic rays in RAMSES

## RAMSES SNO meeting - November 2025

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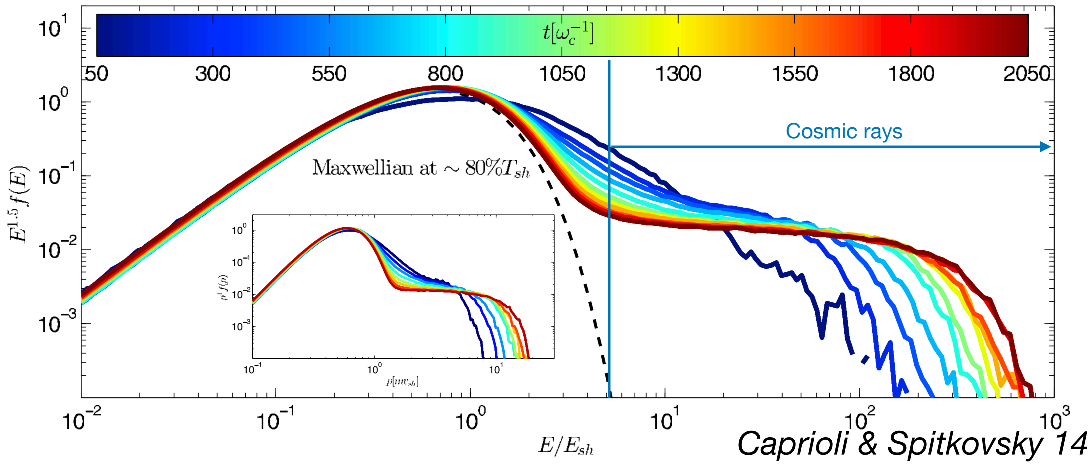
# Context and motivation

**Current modeling of CRs**

- In RAMSES, CRs are modeled according to a « grey approach »
- But cooling processes and diffusion depend on momentum

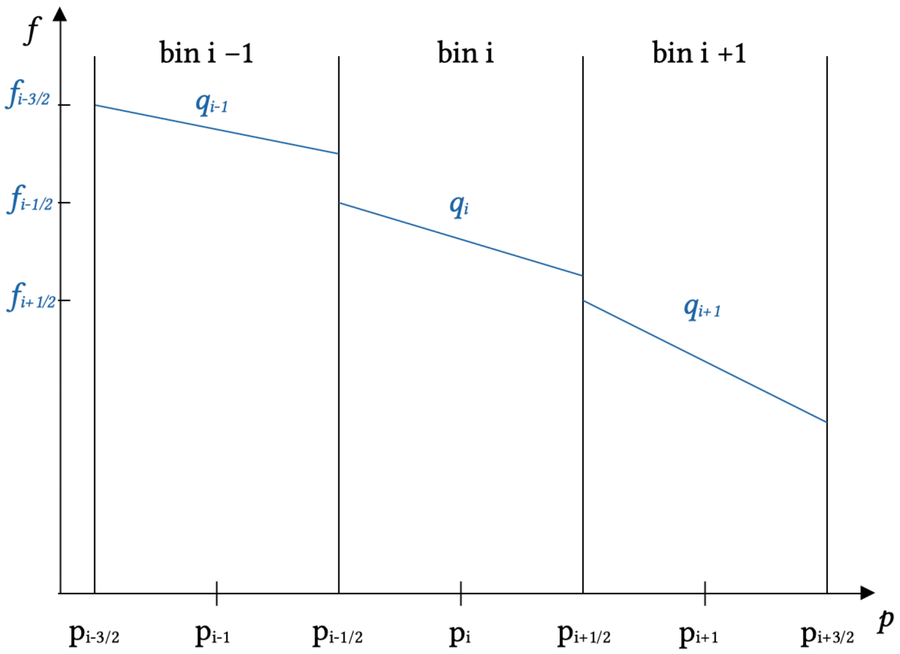



Caprioli & Spitkovsky 14

- To better understand the dynamics of CRs within galaxies, we propose to model the momentum distribution function $f(p, t)$ and follow its evolution

# Numerical scheme

**Based on Girichidis et al. (2020)**




- Piece-wise power law distribution function

$$f(p) = f_{i-1/2}\left(\frac{p}{p_{i-1/2}}\right)^{-q_i}$$

- Number density $n$

$$n_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2\, f(p)dp$$

- Energy density $e$

$$e_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2\, T(p)f(p)dp$$

**Evolution of** $(f_{i-1/2}, q_i)$ <=> **Evolution of** $(e_i, n_i)$

# Spectral method

## Equations on distribution function $f$

$f(x, p, \mu, t) = f_0(x, p, t) + 3\mu f_1(x, p, t)$ with $\mu = \vec{p}\,.\,\vec{b}$ the CR particle pitch angle

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Equation evolving the isotropic part of $f$: $$\frac{\partial f_0}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_0) + \vec{\nabla}\,.\,(\mathrm{v}\vec{b} f_1) = \frac{1}{p^2}\frac{\partial}{\partial p}\left[p^2 b(p) f_0\right] + j_0$$

Equation evolving the anisotropic part of $f$: $$\frac{\partial f_1}{\partial t} + \vec{\nabla}\,.\,(\vec{u} f_1) + \mathrm{v}\vec{b}\,\vec{\nabla}\left(\frac{f_0}{3}\right) = -\left[\bar{D}_{\mu\mu} f_1 + \bar{D}_{\mu p}\frac{\partial f_0}{\partial p}\right] + j_1$$

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$\vec{u}$ : gas velocity

v : CR particle velocity

$\bar{D}_{\mu\mu}$: scattering rate

$\bar{D}_{\mu p}$: streaming transport

$j_{0,1}$: CR injection

$$b(p) = \underset{\text{Radiative losses}}{b_l} + \underset{\text{Adiabatic changes}}{p \frac{\vec{\nabla} \,.\, \vec{u}}{3}} + \underset{\text{Streaming losses}}{D_{p\mu} \frac{f_1}{f_0}} + \underset{\text{Fermi II acceleration}}{\frac{D_{pp}}{f_0} \frac{\partial f_0}{\partial p}}$$

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# Spectral method

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from $f_0$ : $\begin{cases} n_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 f_0(p)dp \\ e_i = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} T(p) p^2 f_0(p)dp \end{cases}$

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from $f_1$: $$\begin{cases} F_i^n = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} f_1(p)dp \\ F_i^e = 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} T(p)p^2 \mathrm{v} f_1(p)dp \end{cases}$$

# Spectral method

**Equations on** $n_i$, $F_i^n$, $e_i$, $F_i^e$

$$\frac{\partial n_i}{\partial t} + \vec{\nabla} \,.\, (\vec{u} n_i) + \vec{\nabla} \,.\, \vec{F}_i^n = \boxed{\left[4\pi p^2 b(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}}} + j_{0,i}^n$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^n}{\partial t} + \vec{b}\vec{b} \,.\, \vec{\nabla}\left(\frac{n_i}{3}\right) = -\frac{1}{3\kappa_i^n}\left[\vec{F}_i^n - \frac{q_i}{3}\bar{u}_{\mathrm{A}} n_i\right]$$

$$\frac{\partial e_i}{\partial t} + \vec{\nabla} \,.\, (\vec{u} e_i) + \vec{\nabla} \,.\, \vec{F}_i^e = \boxed{\left[4\pi p^2 b(p) T(p) f_0\right]_{p_{i-1/2}}^{p_{i+1/2}} - 4\pi \int_{p_{i-1/2}}^{p_{i+1/2}} p^2 \mathrm{v} b(p) f_0 \mathrm{d}p} + j_{0,i}^e$$

$$\frac{1}{\mathrm{v}^2}\frac{\partial \vec{F}_i^e}{\partial t} + \vec{b}\vec{b} \,.\, \vec{\nabla}\left(\frac{e_i}{3}\right) = -\frac{1}{3\kappa_i^e}\left[\vec{F}_i^e - \frac{q_i}{3}\bar{u}_{\mathrm{A}} e_i\right] .$$

- Terms transferring energies between momentum bin
- $\kappa_i^e = \kappa_i^n = \kappa_i$ and diffusion coefficient depends on $p$: $\kappa(p) \propto p^{0.5}$



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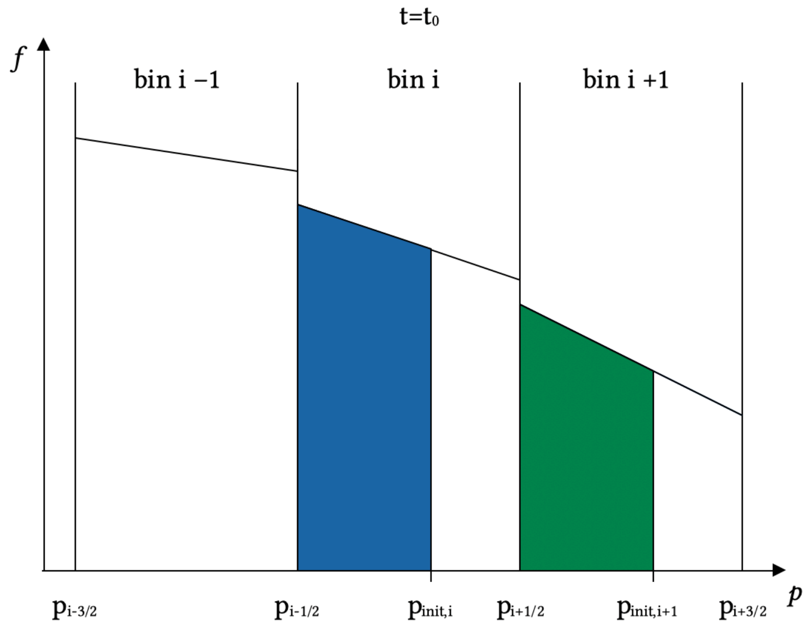
# Spectrum time evolution

## Update in momentum

- Flows in *e* and *n* between bins

$$n_i(t+\Delta t) = n_i(t) - \Delta n_{i-1/2} + \Delta n_{i+1/2}$$

$$e_i(t+\Delta t) = \left(e_i(t) - \Delta e_{i-1/2} + \Delta e_{i+1/2}\right)(1 - R_i \Delta t)^{-1}$$



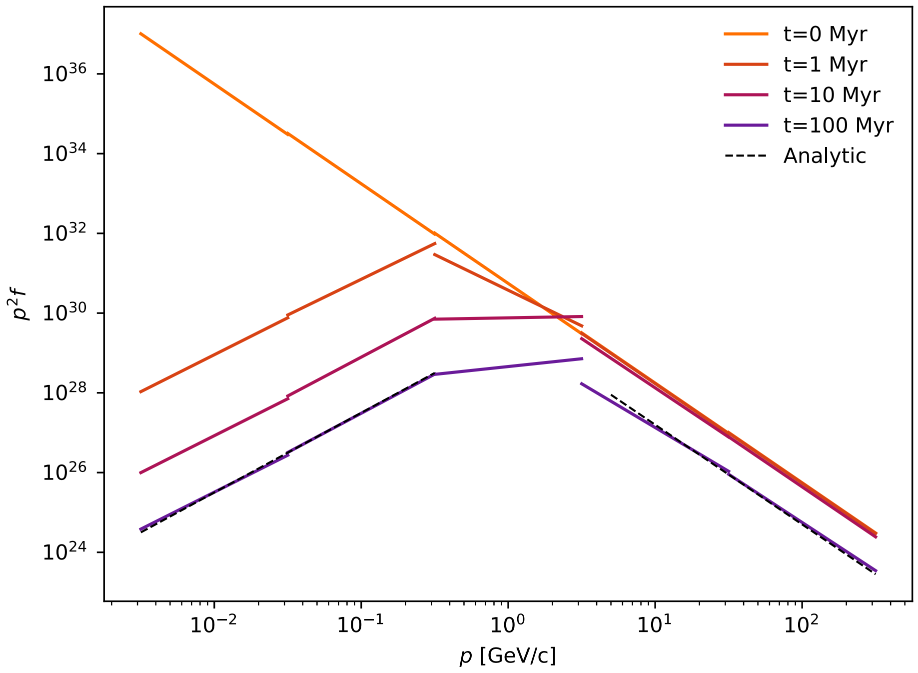

$$\Delta n_{i-1/2} = \int_{p_{i-1/2}}^{p_{init,i}} 4\pi p^2 f dp \qquad \Delta e_{i-1/2} = \int_{p_{i-1/2}}^{p_{init,i}} 4\pi p^2\, T(p) f dp$$

# Radiative losses

## Application: Free cooling test

- Coulomb losses: dominate at low momentum ($p < 1$ GeV/c)

- Hadronic losses: dominate at high momentum ($p > 1$ GeV/c)

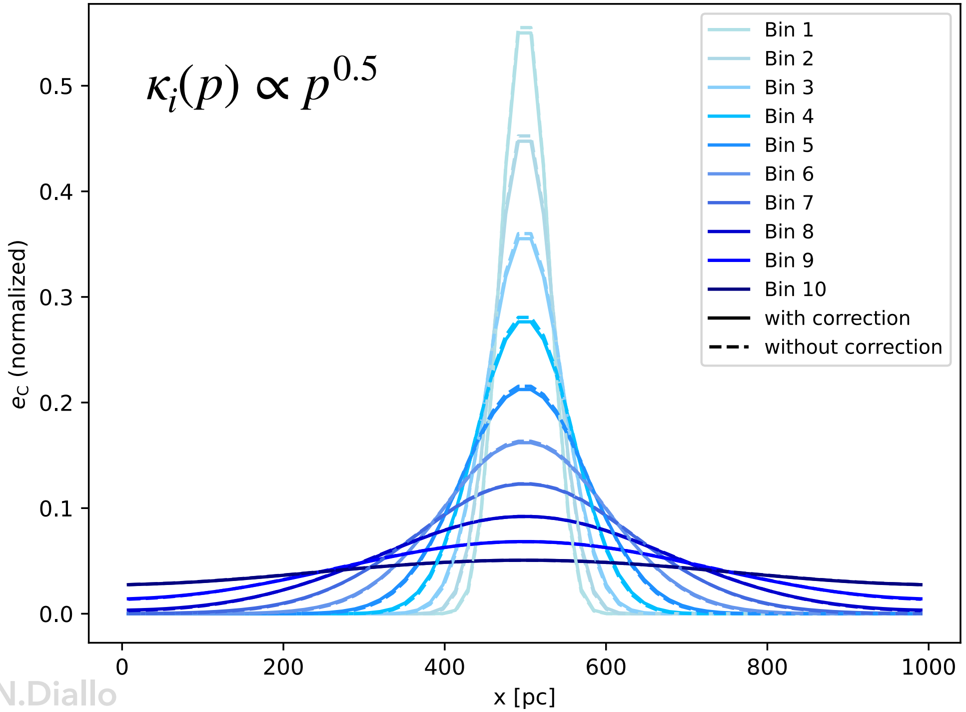


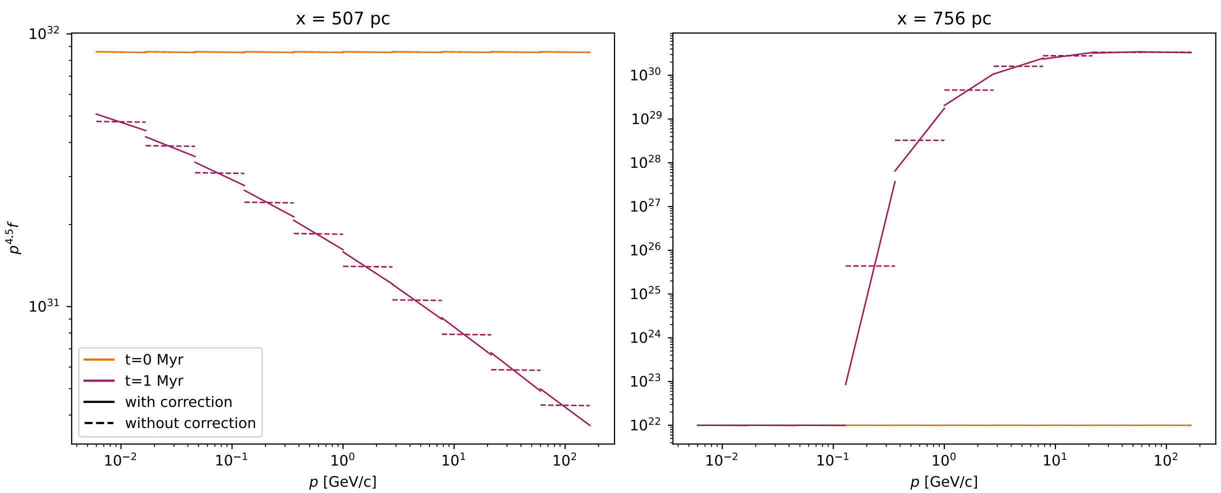

For $N = 10$ bins and $q = 4.5$

# Diffusion

## Application: 1D diffusion test

- High-momentum CRs diffuse faster than low-momentum CRs
- Correction on the diffusion coefficient

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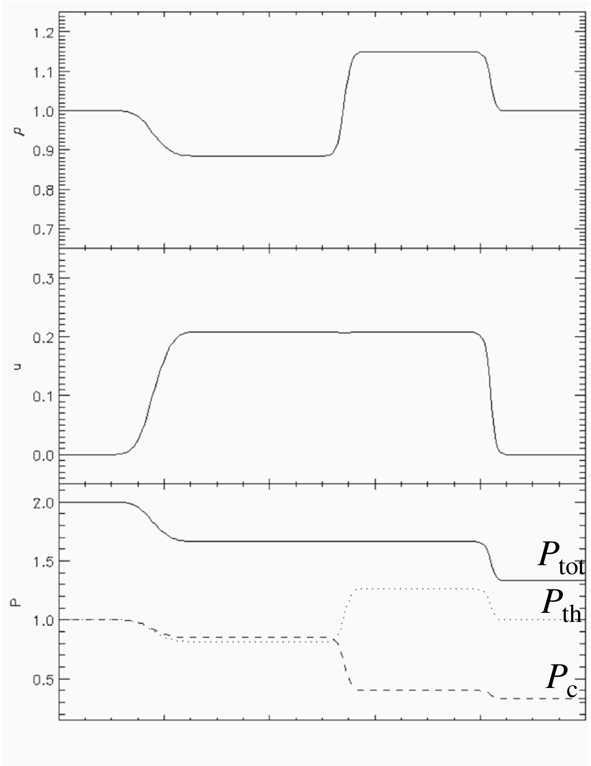
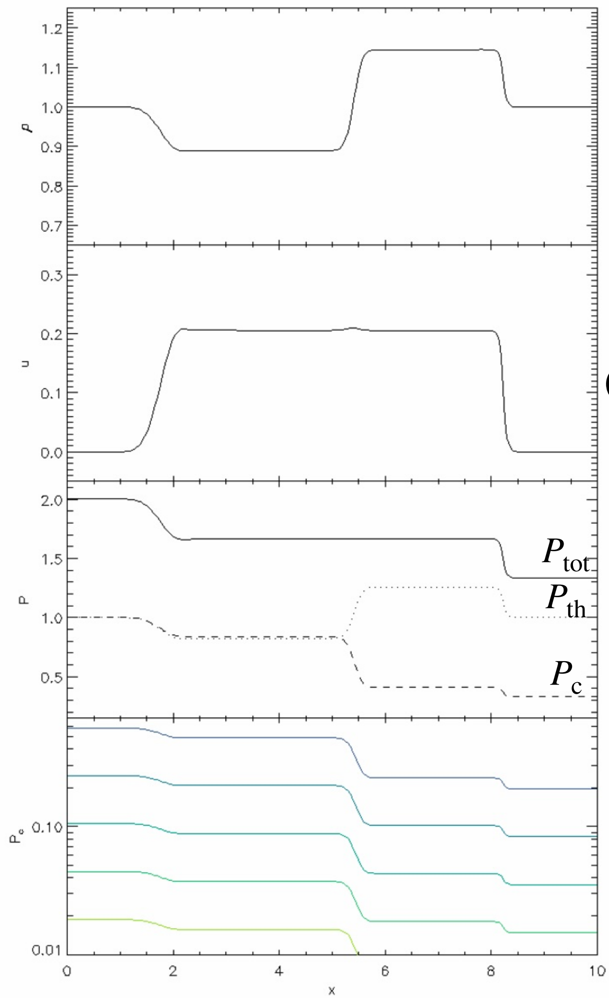
# Shock tube

CR two-moment (Rosdhal+25)




CR spectral method

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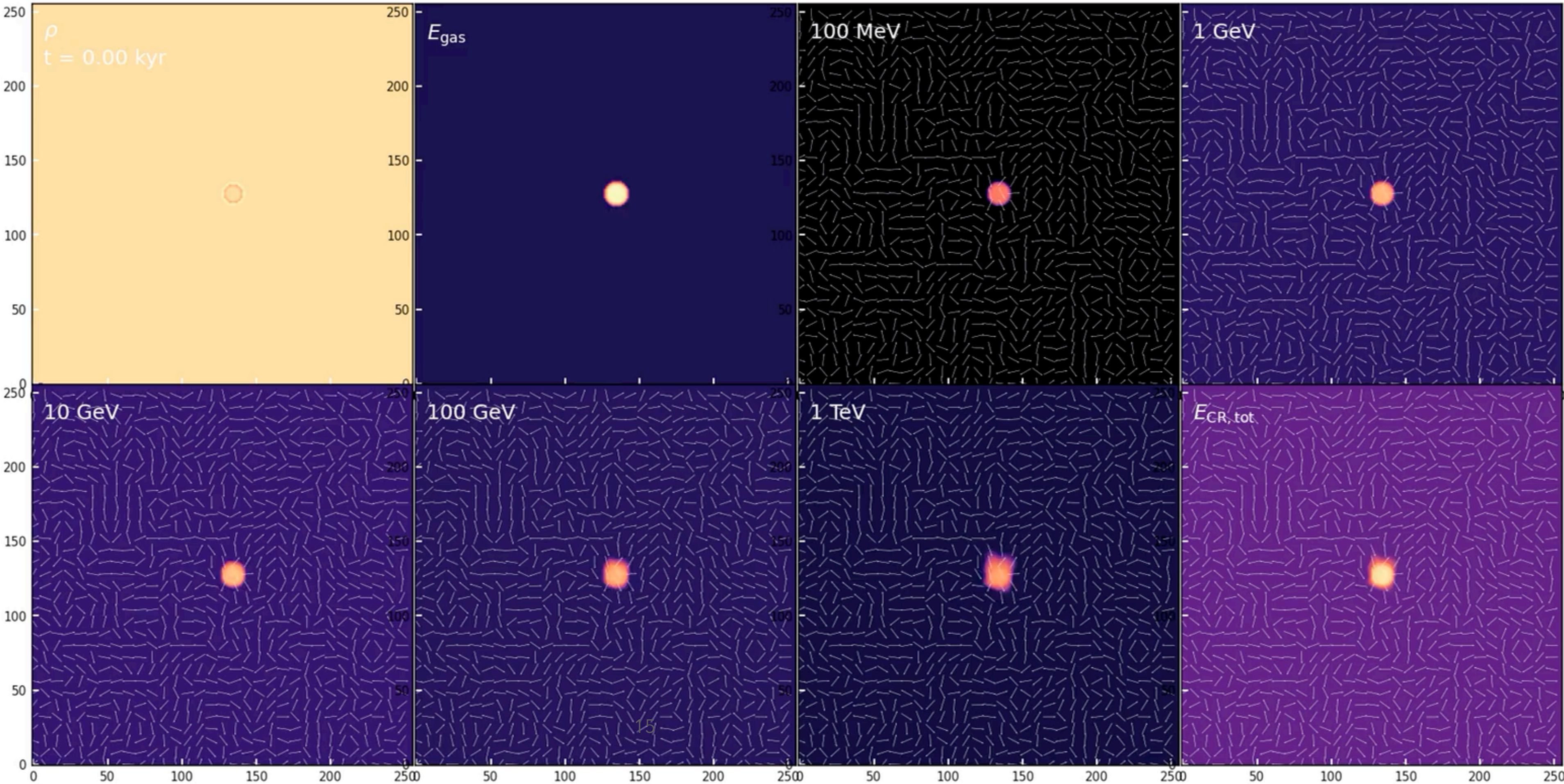
# Supernova remnant

$$\kappa_i(p) = 3 \times 10^{25} \left(\frac{p}{1\mathrm{GeV}}\right)^{0.5} \ \mathrm{cm^2\,s^{-1}}$$

$$E_{\mathrm{SN}} = 10^{51}\,\mathrm{erg}$$

$$n_{\mathrm{gas}} = 10\,\mathrm{cm^{-3}}$$

$$\langle B \rangle \approx 0.025\ \mu\mathrm{G}$$

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# Summary

- Development and implementation of the spectral method for CRs in RAMSES
- Different idealized tests validate this method
- Simulation of supernovae

*Next step*: study momentum deposition of SNe with CRs (Rodriguez Montero et al. 2022)

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Currently looking for
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