Two-moment Cosmic Rays in RAMSES

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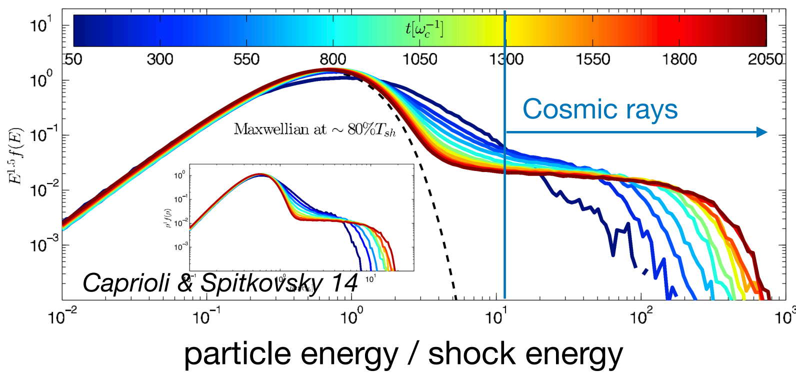
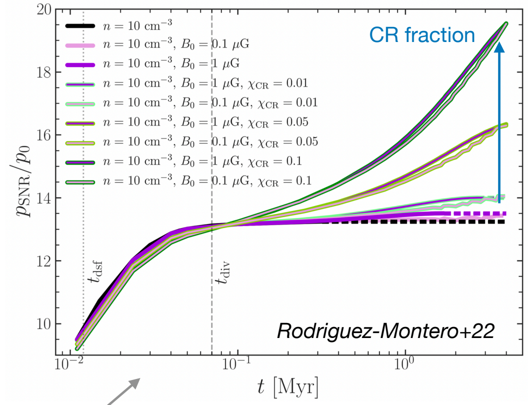
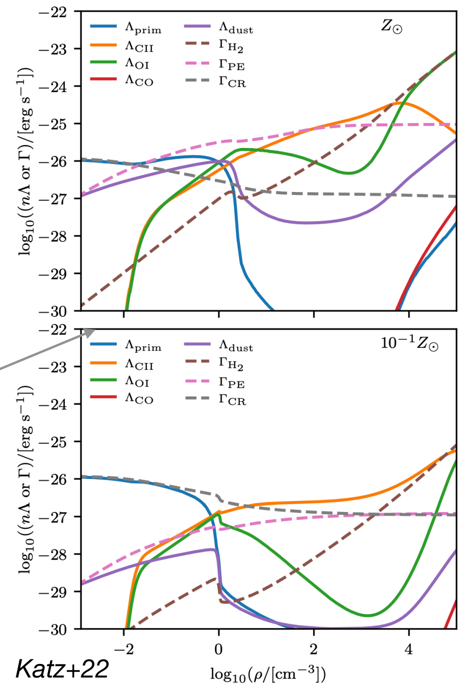
Why should we care about cosmic rays?

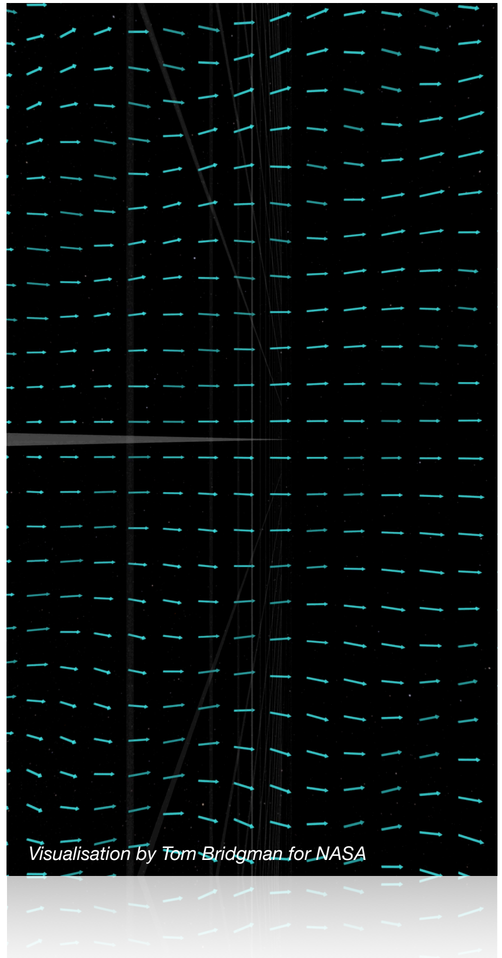




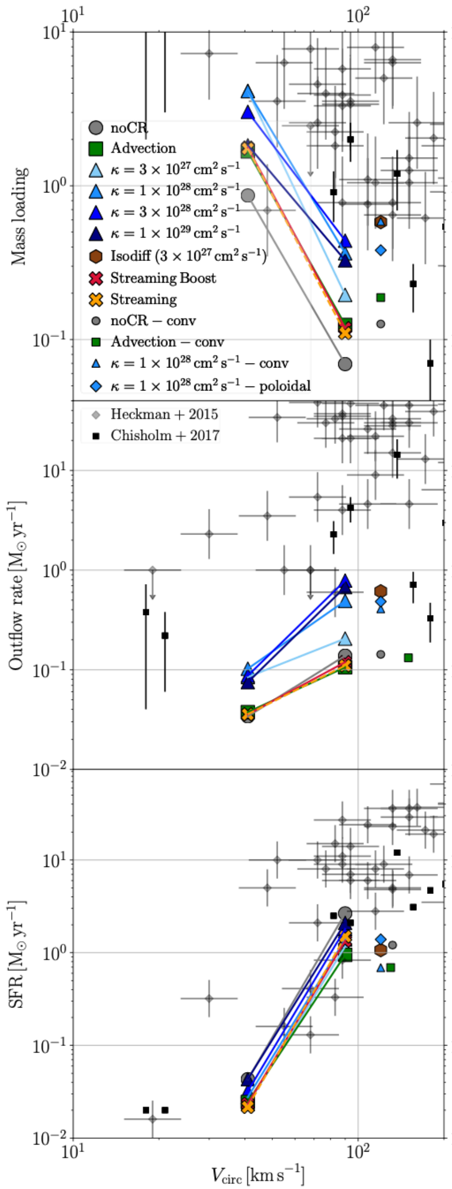


- **Equipartition of energies** (kinetic ~ thermal ~ magnetic ~ cosmic rays) in galaxy formation problems: intra-cluster medium, active galactic nuclei jets, galactic winds, interstellar medium
- As a relativistic population of particles **their adiabat and losses are different from that of the gas**
- **Diffusion** is a key aspect of cosmic ray transport
- Cosmic rays are **produced at shocks**: supernovae, jets, cosmic infall
- More momentum in SN explosions
- Important heating mechanism in the diffuse ISM
- Sets the electron fraction at high cloud densities due to CR ionisation losses

*CR particle individual speed is ~c, however, the **CR fluid velocity** is $\{u_{gas}+u_A$ + a diffusion speed$\}$ due to efficient CR scattering with Alfvén waves*

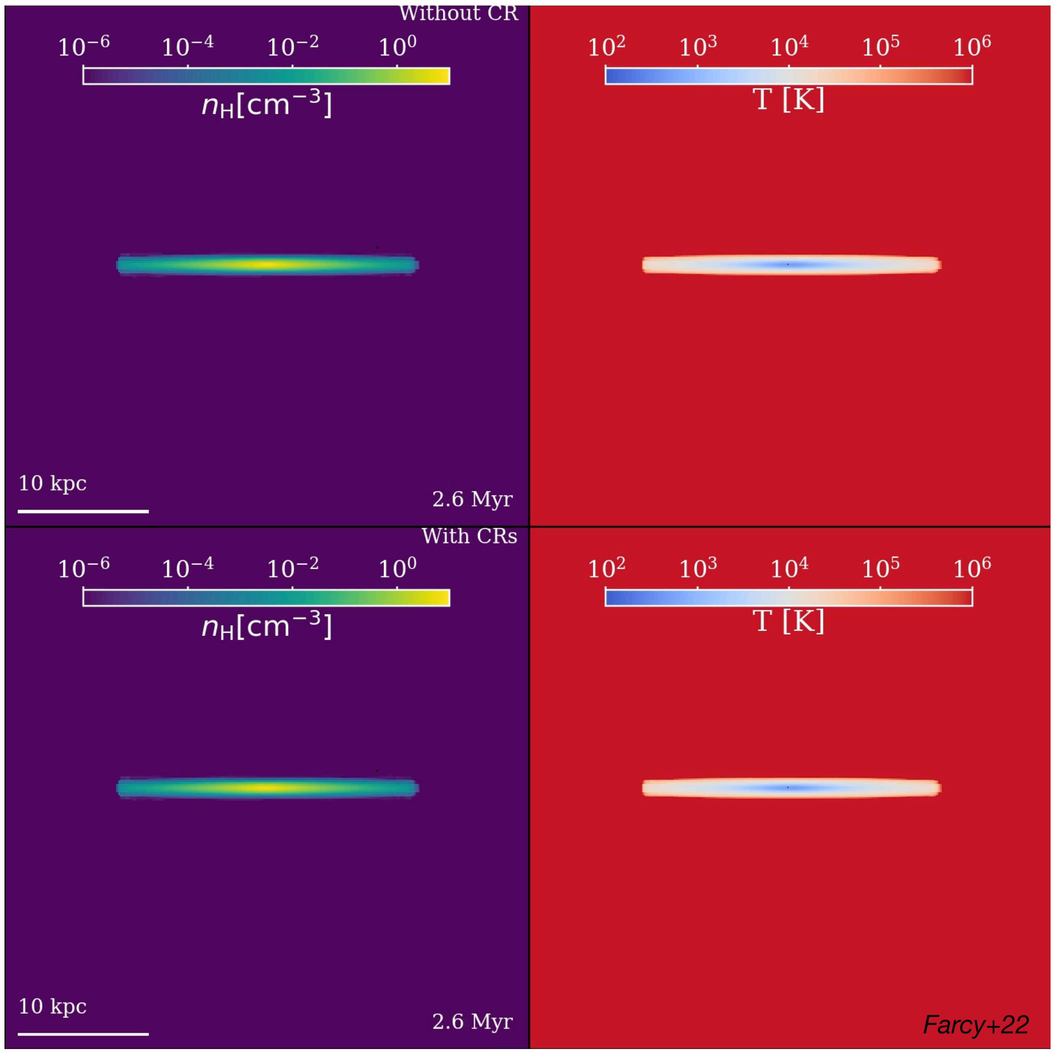


CR-driven large-scale galactic winds



Dashyan & Dubois 20

- Better match to observations w/ CRs
- More mass removed by galactic winds
- Winds are CR pressure-dominated
- Winds are faster and denser, colder
- CRs reduce the amount of dense SF gas



Farcy+22

Cosmic ray magneto-hydrodynamics

1-moment already in RAMSES (Dubois & Commerçon 2016, 2019)

$$\frac{\partial \rho}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}) = 0,$$

$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \boldsymbol{\nabla} \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathsf{P}^*) = \kappa^{-1}[\boldsymbol{F}_c - \boldsymbol{v} \cdot (E_c \mathsf{I} + \mathsf{P}_c)],$$

$$\frac{\partial E}{\partial t} + \boldsymbol{\nabla} \cdot [(E + P^*)\boldsymbol{v} - \boldsymbol{B}(\boldsymbol{B} \cdot \boldsymbol{v})] = -(\boldsymbol{v} + \boldsymbol{v}_s) \cdot (\boldsymbol{\nabla} \cdot \mathsf{P}_c),$$

$$\frac{\partial \boldsymbol{B}}{\partial t} - \boldsymbol{\nabla} \times (\boldsymbol{v} \times \boldsymbol{B}) = 0,$$

$$\frac{\partial E_c}{\partial t} + \boldsymbol{\nabla} \cdot \boldsymbol{F}_c = (\boldsymbol{v} + \boldsymbol{v}_s) \cdot (\boldsymbol{\nabla} \cdot \mathsf{P}_c) + Q,$$

$$\boldsymbol{F}_C = (E_C + \mathrm{P_C})(\boldsymbol{v} + \boldsymbol{v}_s) \quad \leftarrow \text{Advection}$$

$$-\kappa \boldsymbol{b}(\boldsymbol{b} \cdot \nabla \mathrm{E_C}) \quad \leftarrow \text{Diffusion}$$

- 1-moment drawbacks:
  - severe timestepping condition (-> implicit)

    $$\Delta t \ < \ C_{\mathrm{cour}}\,\Delta x^2/\kappa$$

  - overly diffusive
  - do not handle the physical speed-of-light limit

Cosmic ray magneto-hydrodynamics

2-moment, now added to RAMSES: Rosdahl et al. just on arXiv

based on Jiang&Oh (2017) in ATHENA, and similar to Chan et al. (2019) and others

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

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$$\frac{\partial \boldsymbol{B}}{\partial t} - \nabla \times (\boldsymbol{v} \times \boldsymbol{B}) = 0,$$

lab-frame formulation of $\vec{F}_c$

$$\frac{\partial E_c}{\partial t} + \nabla \cdot \boldsymbol{F}_c = (\boldsymbol{v} + \boldsymbol{v}_s) \cdot (\nabla \cdot \mathsf{P}_c) + Q,$$

$$\frac{1}{V_m^2}\frac{\partial \boldsymbol{F}_c}{\partial t} + \nabla \cdot \mathsf{P}_c = -\kappa^{-1}[\boldsymbol{F}_c - \boldsymbol{v} \cdot (E_c \mathsf{I} + \mathsf{P}_c)].$$

$$\vec{\nabla} \cdot (\mathbb{D} E_c) \rightarrow \vec{\nabla} \cdot (\mathbb{I} P_c)$$

$\mathbb{D} = \frac{\mathbb{I}}{3} + ~~(1 - 3\chi)\vec{b}\vec{b}~~$ **P1 approx. of pressure tensor (M1?)**

- Here the **F**$_c$ becomes an extra variable, evolved in each time-step
- V_m is a *maximum* CR speed (or "reduced light speed"), chosen by user at runtime
  - Limits the time-step with the usual Courant condition, i.e.

$$\Delta t \approx \Delta x / V_m$$

  - V_m **must** be larger than the gas speed and *preferentially* larger than the streaming speed and diffusion speed

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$$\frac{\delta \rho \mathbf{v}}{\delta t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*) = \nabla \cdot P_c$$

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$$\frac{\partial \mathcal{U}}{\partial t} + \nabla \mathcal{F}(\mathcal{U}) = 0$$

$$\mathcal{U} = [E_c, F_x, F_y, F_z]$$

Explicit integration for hyperbolic part:

$$\mathcal{U}_l^{n+1} = \mathcal{U}_l^n + \frac{\Delta t}{\Delta x}\left(\mathcal{F}_{l-1/2}^n - \mathcal{F}_{l+1/2}^n\right)$$

Linear interpolation for reconstructed variables with van Leer slope limiter

Intercell flux with HLL:

$$(\mathcal{F}_{\mathrm{HLL}})_{l+1/2}^n = \frac{\lambda^+ \mathcal{F}_l^n - \lambda^- \mathcal{F}_{l+1}^n + \lambda^+\lambda^- \left(\mathcal{U}_{l+1}^n - \mathcal{U}_l^n\right)}{\lambda^+ - \lambda^-}$$

Wave-speeds:

$$\lambda = v_{\mathrm{gas}} + f_R\, V_{\mathrm{m}}$$

Some dark magic, not detailed here…

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$$\frac{\partial \mathcal{U}}{\partial t} + \nabla \mathcal{F}(\mathcal{U}) = 0$$

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Intercell flux with Lax-Friedrich:

$$(\mathscr{F}_{\mathrm{LF}})_{\ell+1/2}^n = \frac{\mathscr{F}_\ell + \mathscr{F}_{\ell+1}}{2} + \frac{\lambda}{2}(\mathscr{U}_\ell - \mathscr{U}_{\ell+1})$$

Cosmic ray magneto-hydrodynamics

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$$\frac{\partial E_c}{\partial t} \cancel{+ \nabla \cdot \boldsymbol{F}_c} = (\boldsymbol{v} + \boldsymbol{v}_s) \cdot (\nabla \cdot \mathsf{P}_c) + Q,$$

$$\frac{1}{V_m^2} \frac{\partial \boldsymbol{F}_c}{\partial t} \cancel{+ \nabla \cdot \mathsf{P}_c} = -\kappa^{-1}[\boldsymbol{F}_c - \boldsymbol{v} \cdot (E_c \mathsf{I} + \mathsf{P}_c)].$$

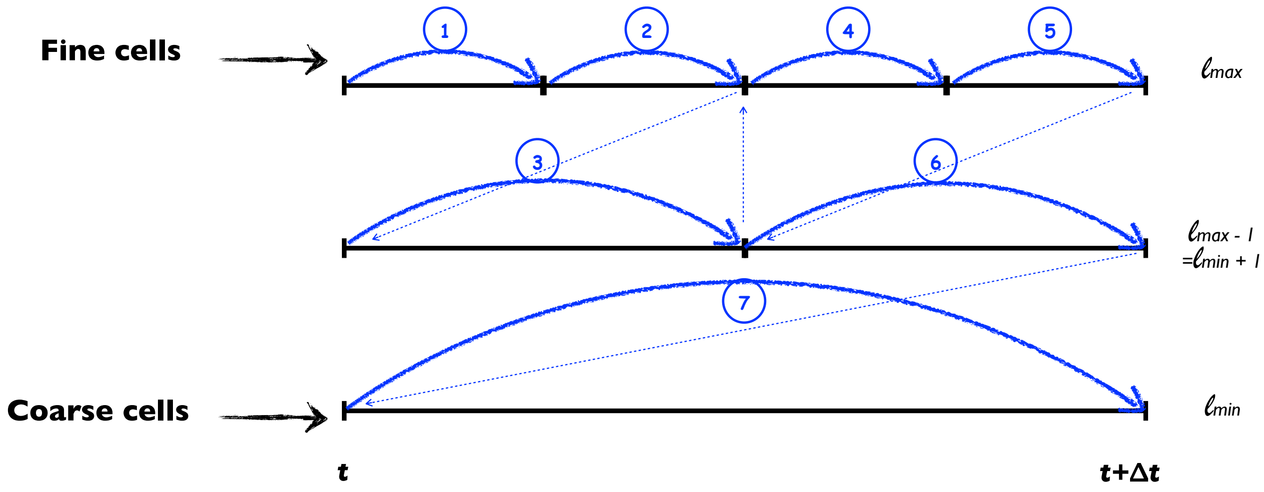
Source terms

- Advection and source terms are ‘operator’ split
- κ is defined in the coordinate-system of **B** $\kappa = [\kappa_{||}, \kappa_{\perp}, \kappa_{\perp}]$
- so must rotate **F**, v, v_s onto **B**, solve, and rotate back
- Implicit time integration for coupled source terms

Adding 2mom CRs to RAMSES

Hydro only

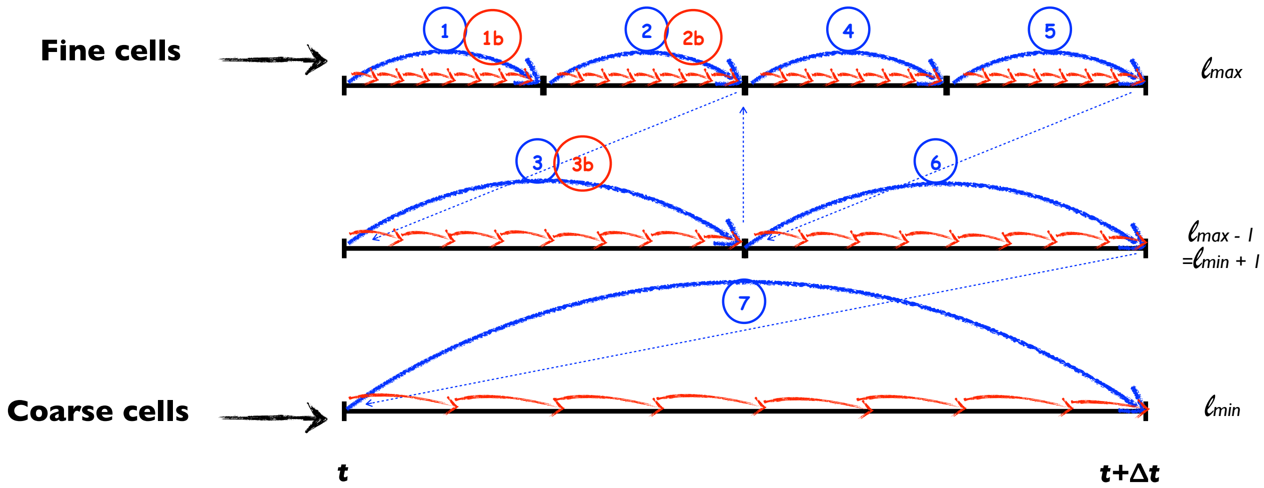
- Two fine steps for every coarse steps, starting at finest level



Adding 2mom CRs to RAMSES

Hydro + CRs

- CRs sub-cycled on each level
- Almost all the additions are in cr_godunov_fine.f90 and cr_flux_module.f90
- Call to crmom_step inside the amr_step routine
- CR sub-cycling precludes exact CR energy conservation across refinement boundaries



Variable speed of light

- We must have $V_{\mathrm{m}} > c_{\mathrm{MHD}}$
- It's difficult to know in advance what will be the maximum c_{MHD}, so we use a variable light-speed, setting in every MHD step in every refinement level:

$$V_{\mathrm{m}} = N\, c_{\mathrm{MHD}}$$

- For the tests, we typically need $N \gtrsim 10$
- But for isolated galaxy test, the results are converged with $N \gtrsim 3$

Tests

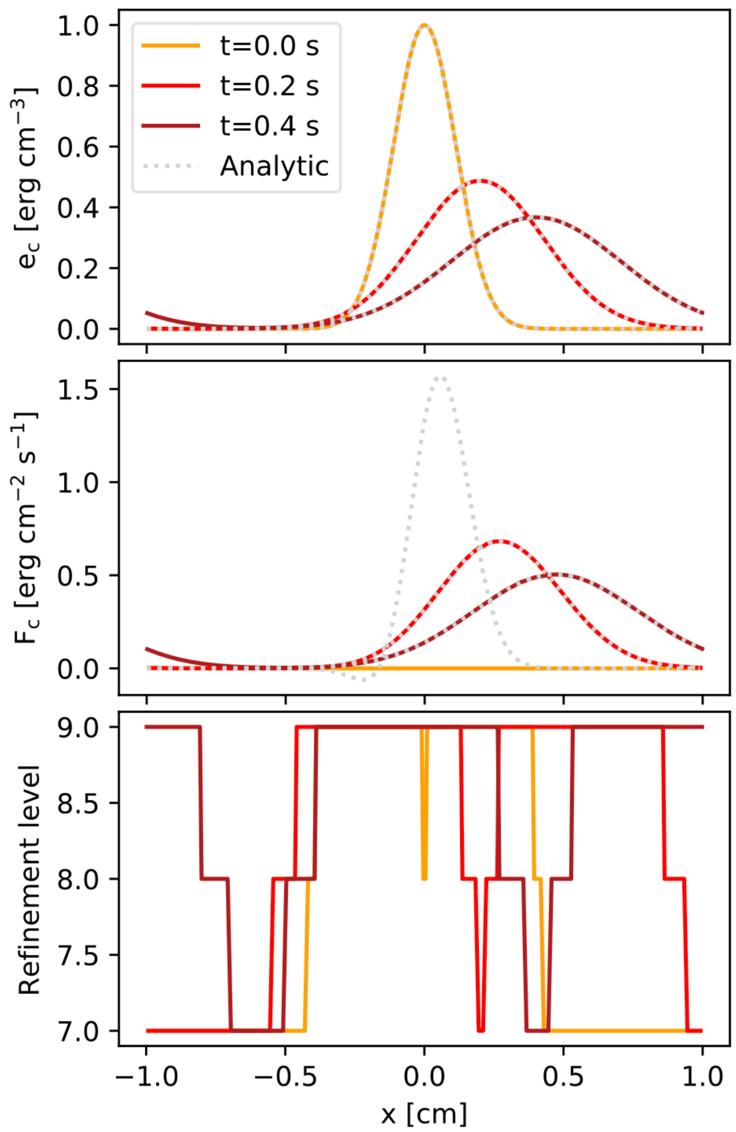
mostly taken from Jiang & Oh (2017)

Tests

Advection + Diffusion in 1d

- Diffusion can be compared to analytic expression, in dotted curves

- With AMR and moving gas
- 👍

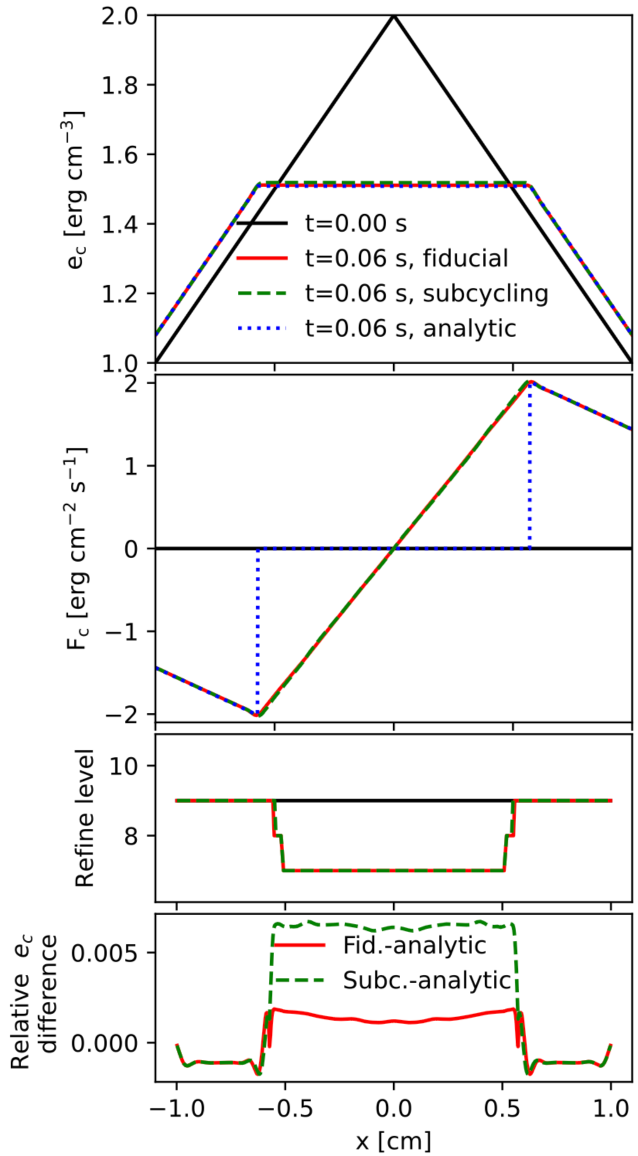


Tests
Pure streaming in 1d

- CRs stream down their own gradient of CR energy
  $u_s = -u_A \mathrm{sign}\left(\vec{b} \,.\, \vec{\nabla} e_c\right)$

- At CR energy extrema u_s becomes discontinuous as they flow in opposite directions due to streaming

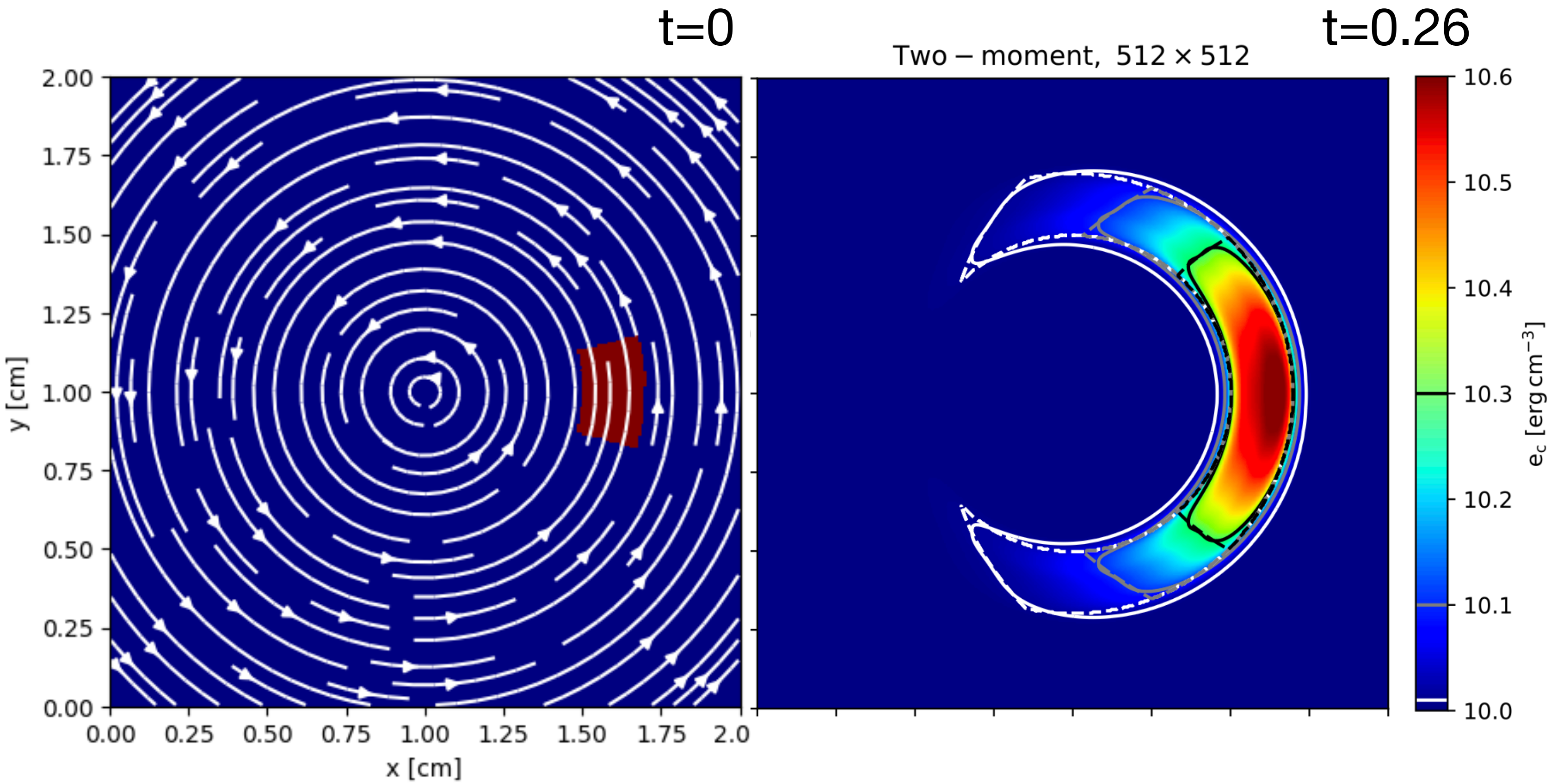
- 👍



Tests

Anisotropic diffusion in 2D

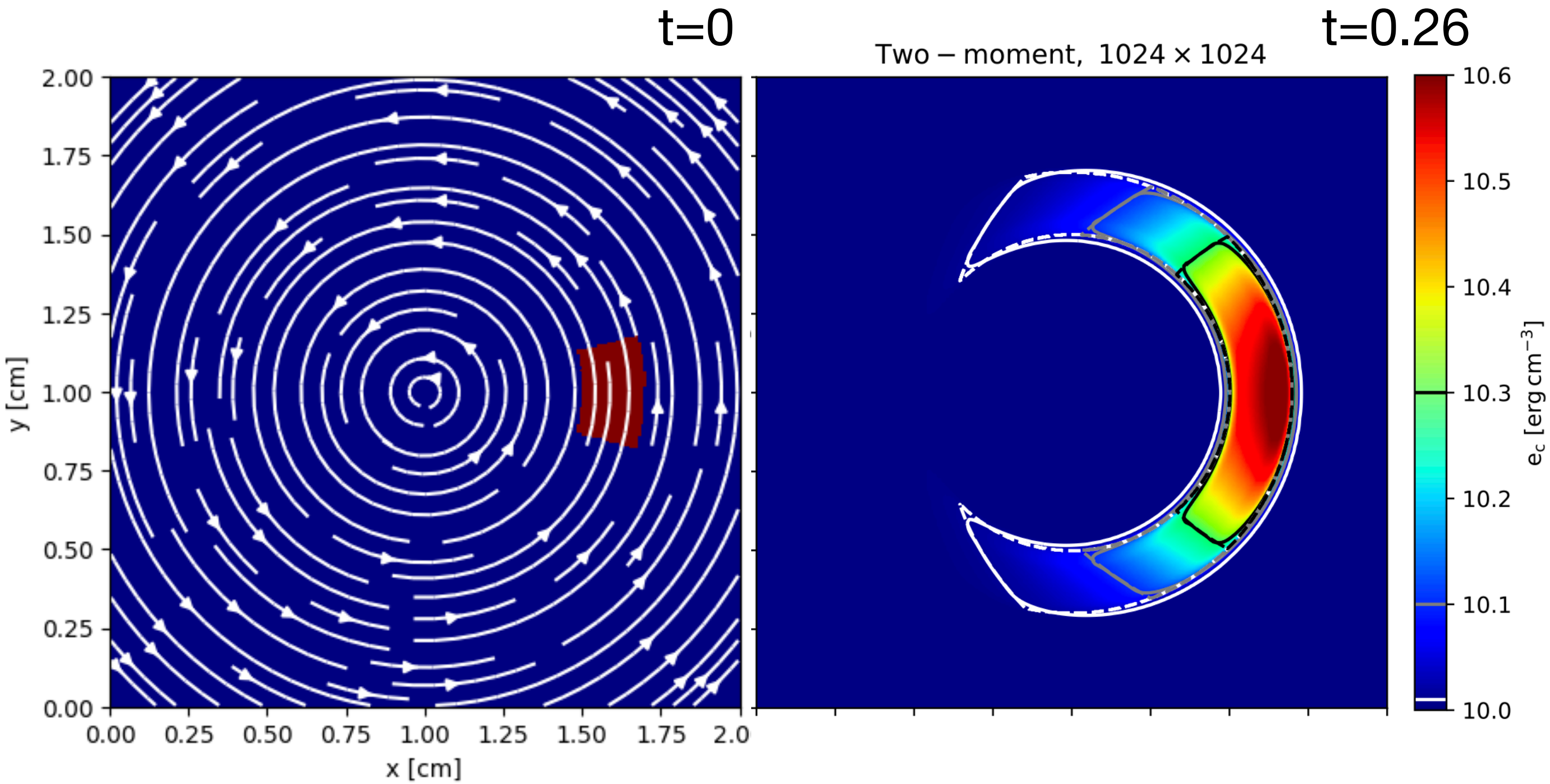
- 512^2 resolution
- Diffusion only



Tests

Anisotropic diffusion in 2D

- 1024^2 resolution
- Diffusion only



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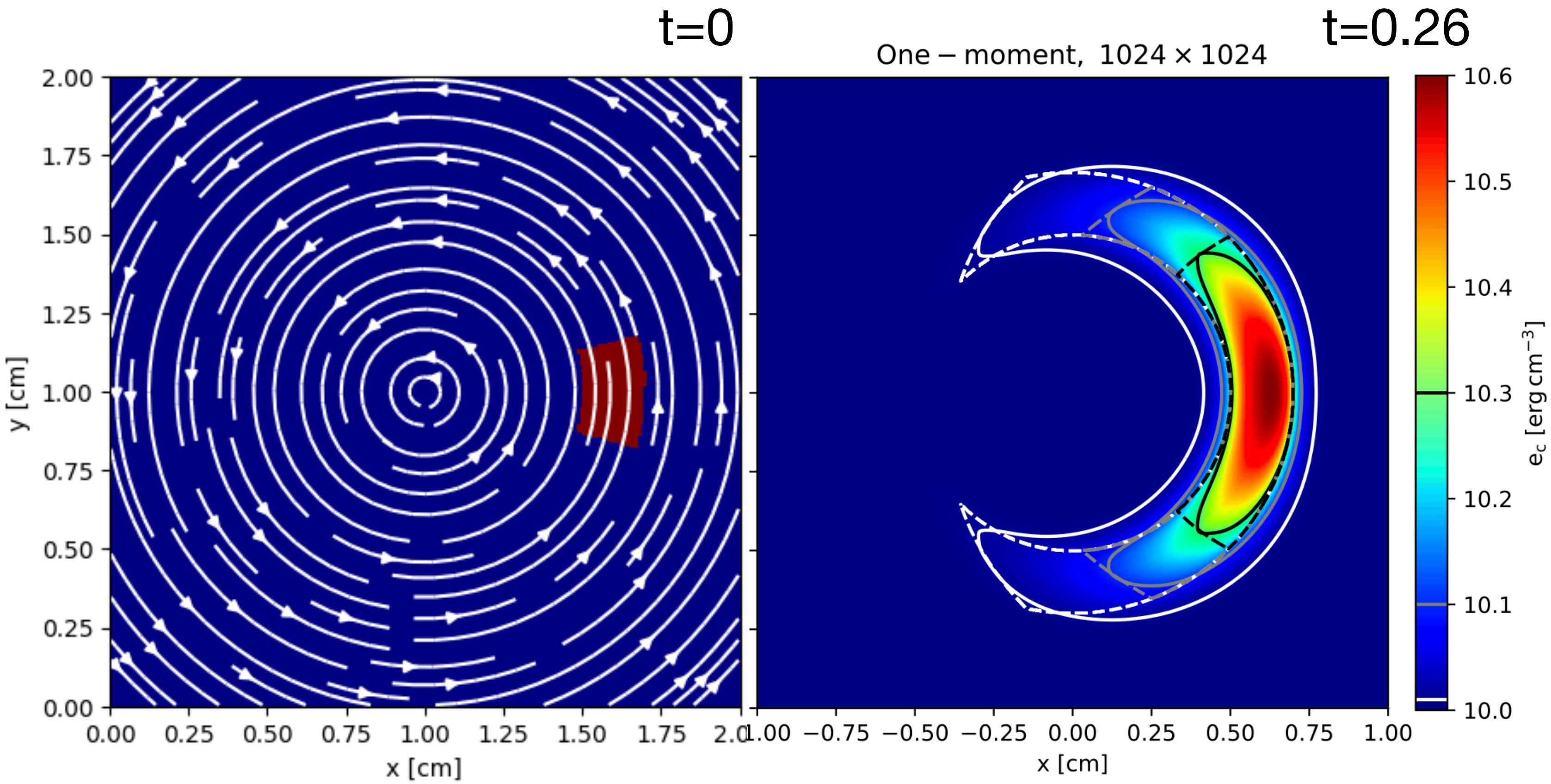
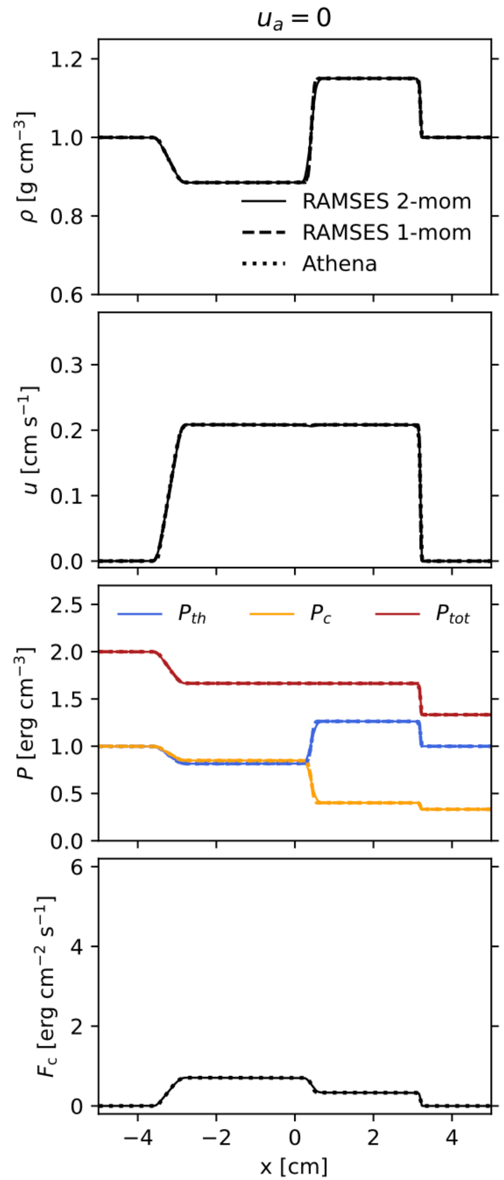


Tests

1D shock tube

- SOD tube generated by CR pressure difference
- Here no diffusion, no streaming
- Well behaved and compares well with other methods



Tests

1D shock tube

- SOD tube generated by CR pressure difference
- Here no diffusion, no streaming
- Well behaved and compares well with other methods
- It miserably fails if we follow standard Athena method
- Gas momentum injection rate:

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}) = 0,$$

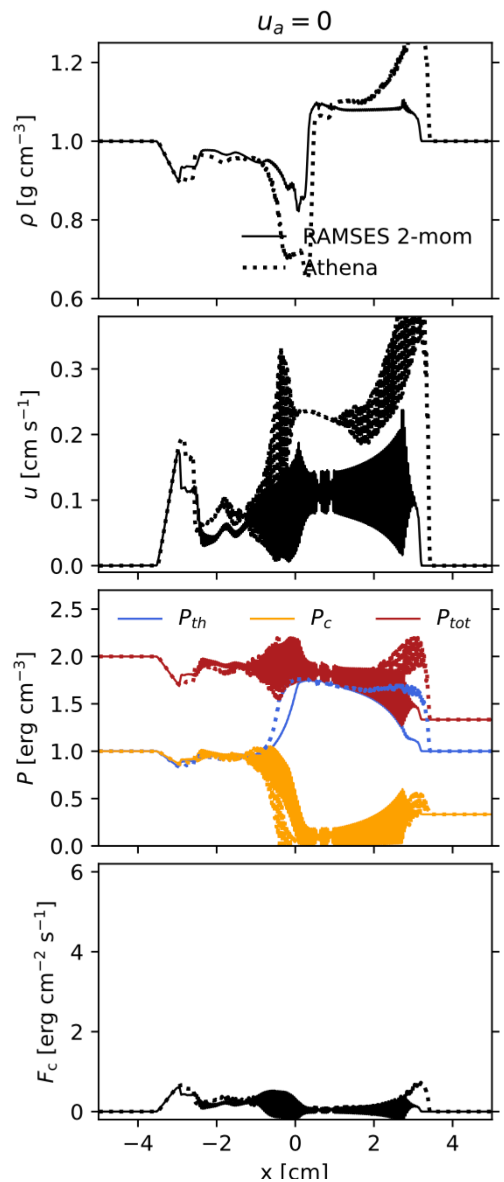
$$\frac{\partial (\rho \boldsymbol{v})}{\partial t} + \nabla \cdot (\rho \boldsymbol{v}\boldsymbol{v} - \boldsymbol{B}\boldsymbol{B} + \mathrm{P}^*) = \kappa^{-1}[\boldsymbol{F}_c - \boldsymbol{v} \cdot (E_c \mathsf{I} + \mathrm{P_c})],$$

Athena

RAMSES

$$\frac{\delta \rho \mathbf{v}}{\delta t} + \nabla \cdot (\rho \mathbf{v}\mathbf{v} - \mathbf{B}\mathbf{B} + P^*) = \boxed{\nabla \cdot P_c}$$

- Also use use Lax-Friedrich flux function instead of HLLE stabilises the solution

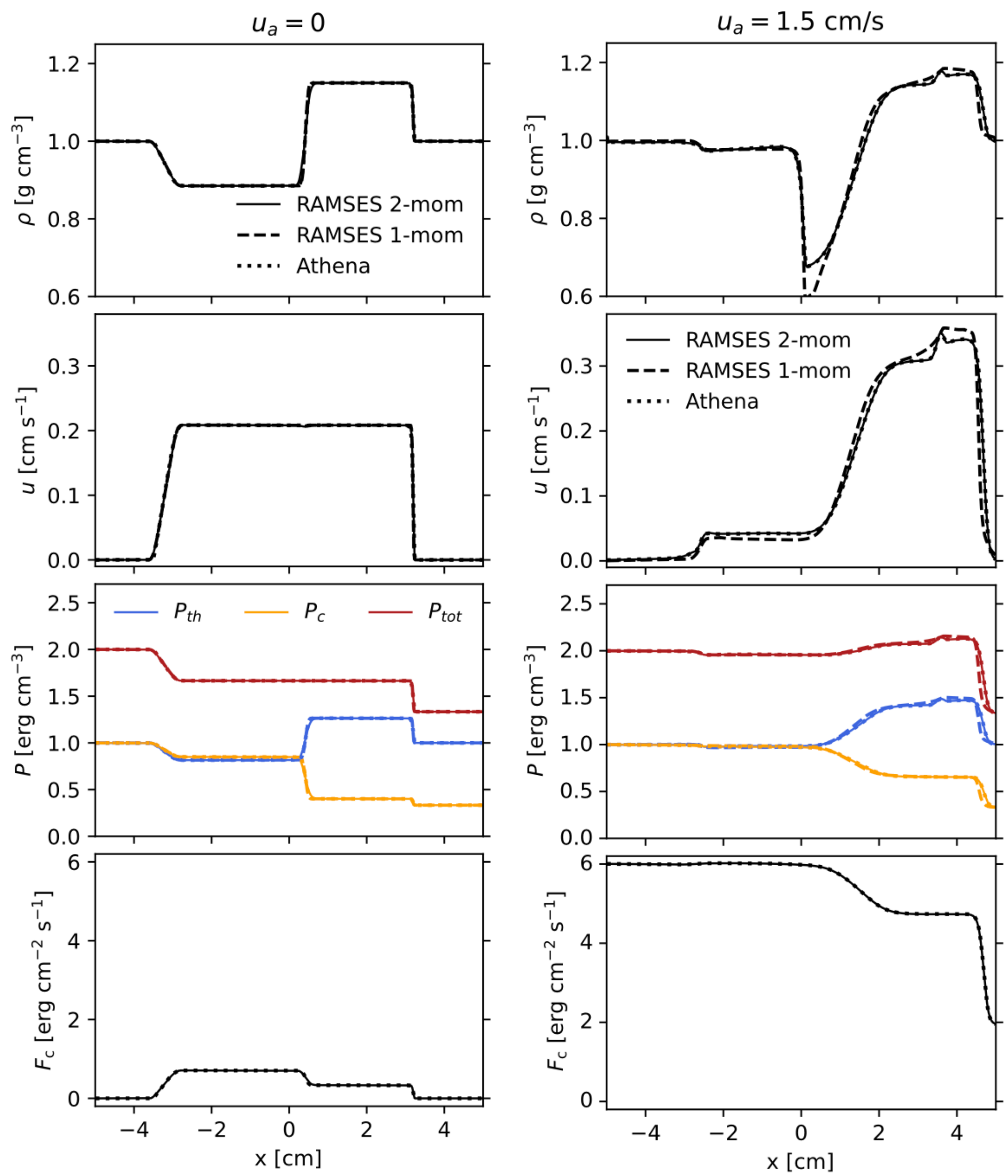


Tests

1D shock tube with streaming

- SOD tube generated by CR pressure difference
- Here no diffusion, streaming with $u_A > u_{shock}$
- Well behaved and compares well with other methods

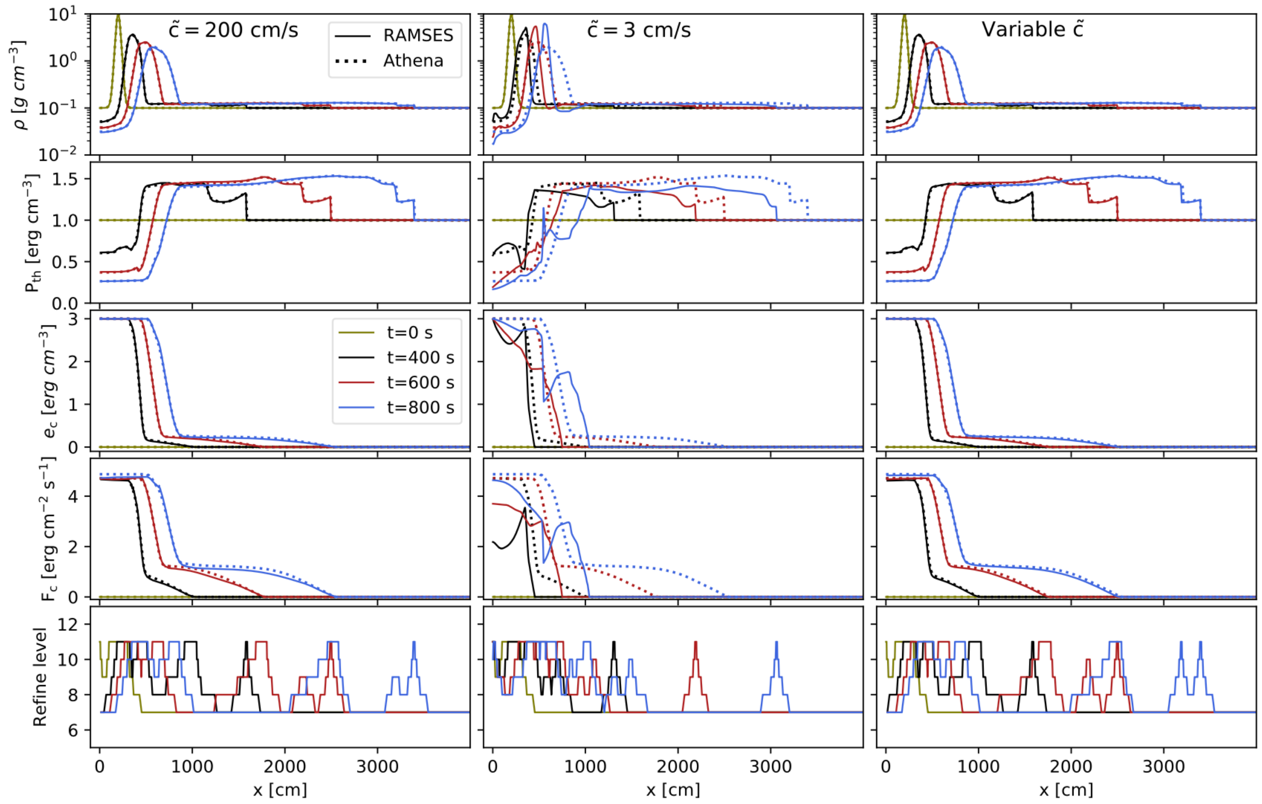
- The density dip at x~0 is due to CR streaming losses at rate $\vec{u}_s . \vec{\nabla} P_c$: CR energy is given to gas energy, but since $\gamma > \gamma_c$, the gradient of total pressure increases locally
- Shock and rarefaction waves now move at $u \pm u_A$ respectively



Tests

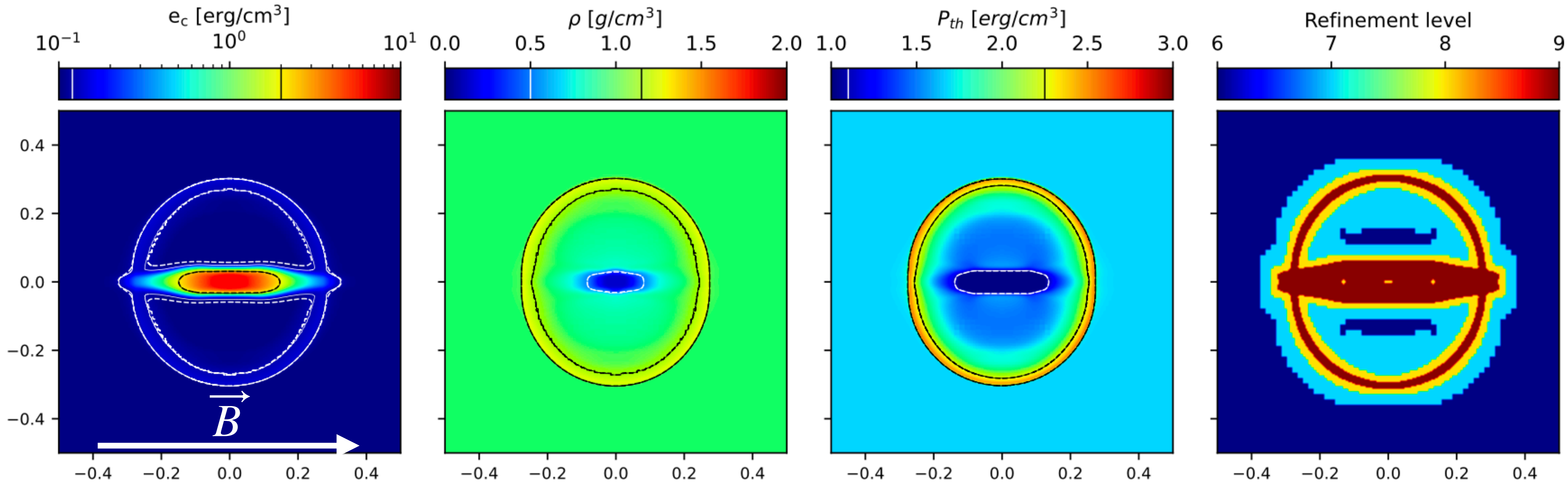
1D cloud with streaming

- $u_A = B/\sqrt{\rho} \sim 0.3 - 3\,\mathrm{cm/s}$
- Bottleneck effect in the cloud where CRs pile up due to reduced streaming velocity
- Compares well with Athena
- **Be careful with reduced speed of light!**



Tests

3D Sedov with anisotropic diffusion



Tests

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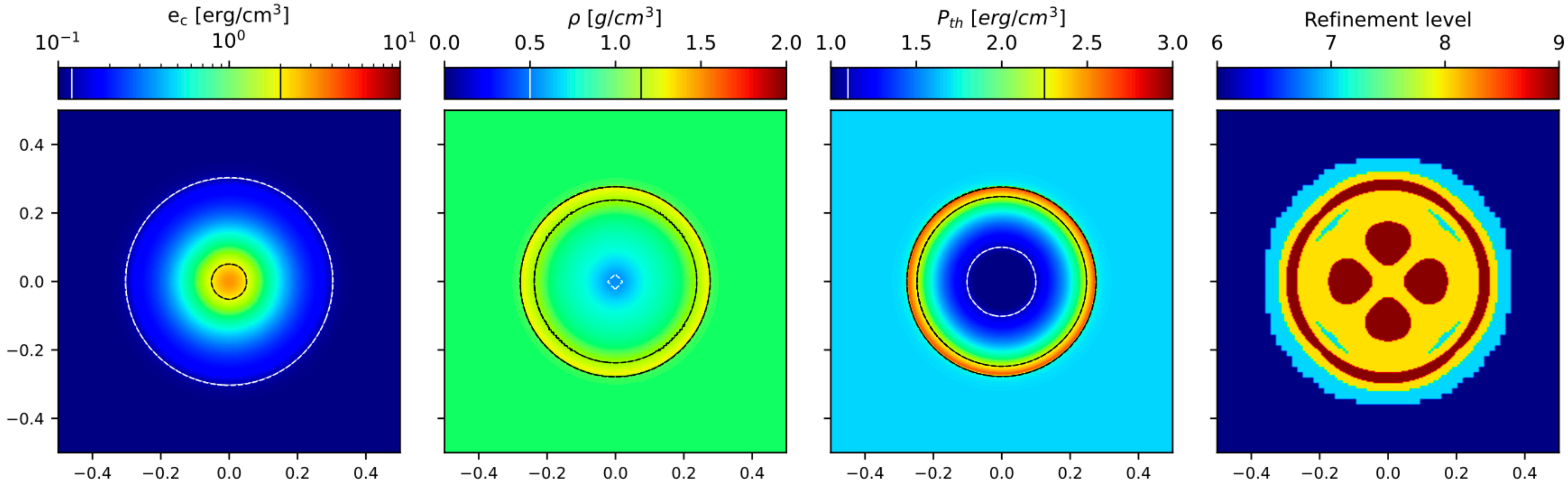
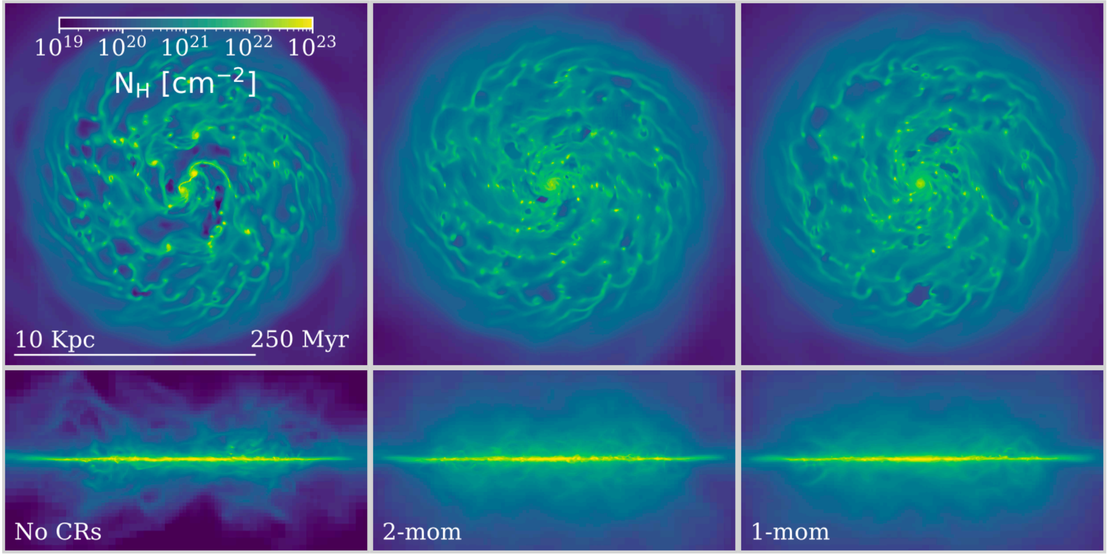
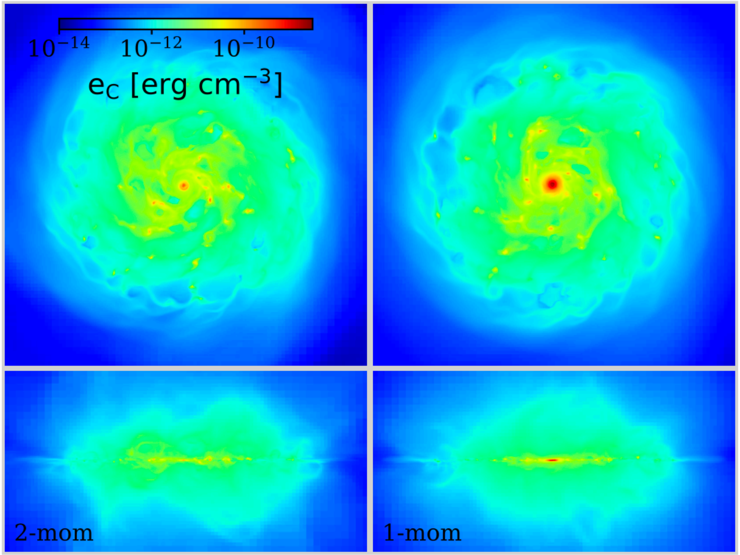


Tests

Isolated “G9” galaxy disk described in Farcy et al. (2022)

- $M_*=10^9\ M_\odot$ galaxy with 18 pc resolution
- $\kappa=3\times10^{28}$ cm²/s
- Compared with 1mom and 2mom in RAMSES





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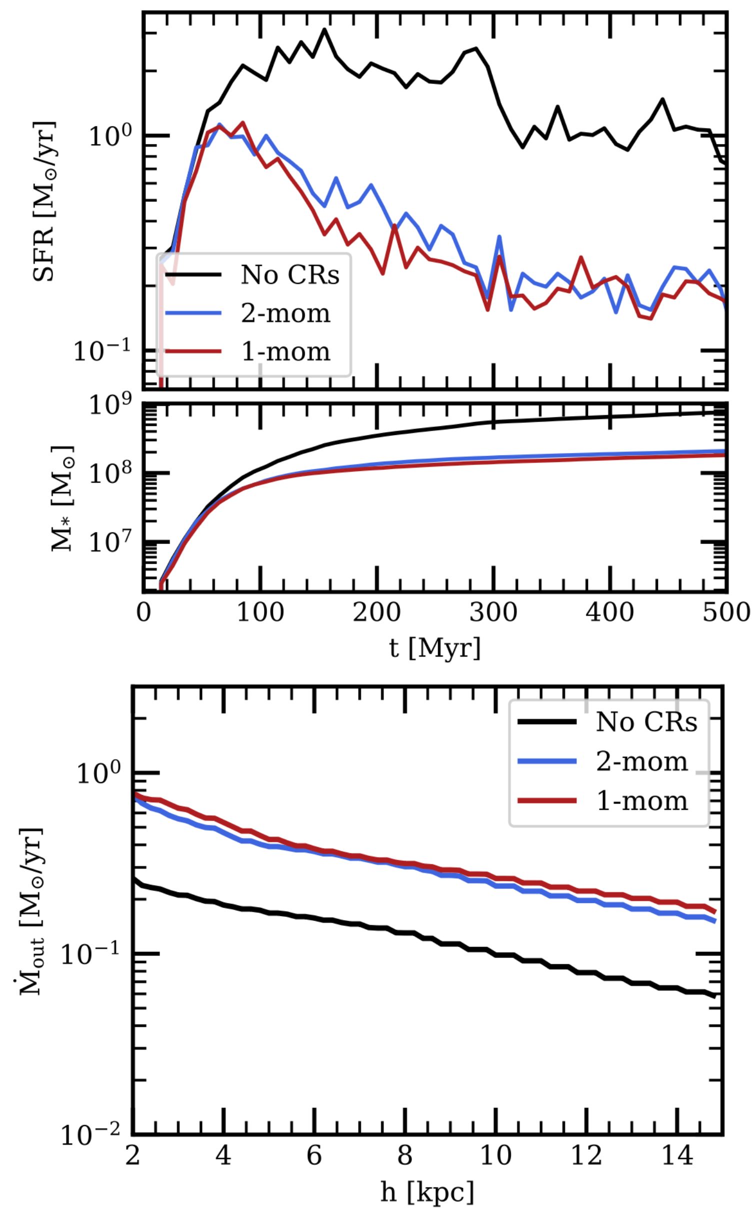
- **In good <u>quantitative</u> agreement**



Conclusions and summary

2-moment Cosmic Rays in RAMSES

- Lower perpendicular numerical diffusion than 1-mom
- Performance:
  - For the constant diffusion galaxy run it is equally good in CR 2-mom and 1-mom (but depends on the speed of light)
  - 2-mom outperforms 1-mom for CR streaming runs (or with variable diffusion)
- Not yet in the public version, but we are happy to share
- The implementation is memory-heavy with 4 variables per CR group!

Recent developments

- **Fluid frame** approach with « NENER » (behaves better in the limit of $c \lesssim V_{\rm mhd}$)
- Now use **M1** (the closure in CR transport is not as critical as in RT)
- Works with **shock finder + injection** (from Dubois+19)
- CR **spectral method** developed by ***Nimatou Diallo*** (IAP)
- Currently implemented in **Dyablo** by ***San Han*** (IAP)
- Variable diffusion coefficient by ***Arturo Núnez-Castiñeyra*** (IAP)
- Can be used for **thermal conduction** with two-temp. ion-elec. (Appendix F of the paper)