

# Cosmological N-body simulations and relativistic ray-tracing

*Yann RASERA (LUX/Université Paris Cité /Paris Observatory/IUF)*

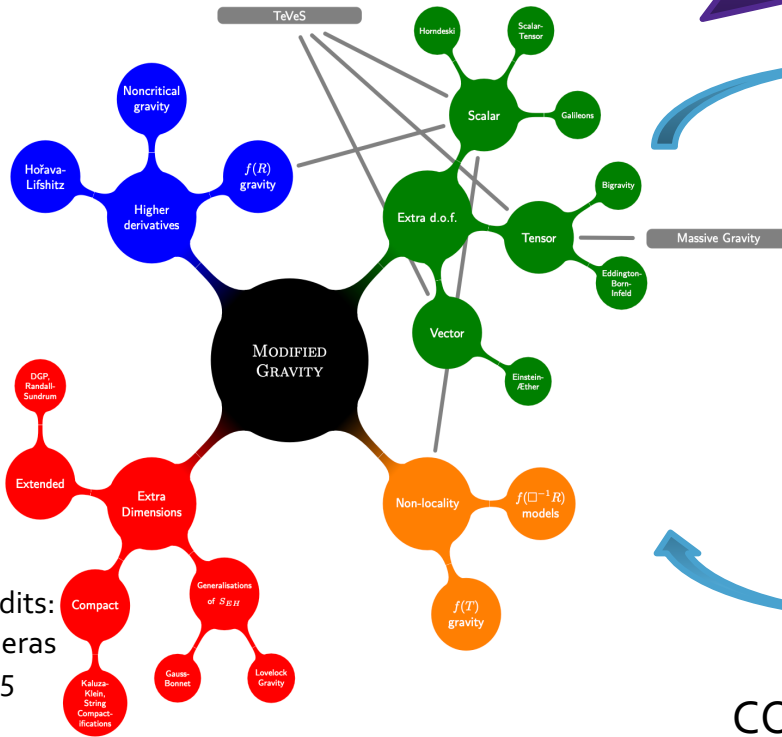
*Work with:*

*Michel-Andrès Breton (CEA), F.Castillo (LUX), M.Corioni (Trieste Univ.), Iñigo Saez-Casares (Milan University), V.Reverdy (LAPP), S.Saga (YITP), A.Taruya (YITP), P.-S. Corasaniti (LUX), J.Allingham (Technion), F.Roy (LUX), T.Pellegrin (LPP), S.Anselmi (INFN), A.Le Brun (LUX)*

# WHAT IS THE NATURE OF GRAVITY AT COSMOLOGICAL SCALES?

## ACCURATE PREDICTIONS

# SIMULATIONS



# ROBUST CONSTRAINTS



## Modified gravity theories

## Large-Scale Structure surveys

# HOW DO WE PROCEED IN THE PROGRACERAY ANR-FUNDED RAMSES-POWERED PROJECT ?

N-BODY SIMULATIONS  
« GENERIC »  
MODIFIED GRAVITY  
MODELS

See  
Talk by  
Himanish  
Ganjoo

HYDRO SIMULATION  
« GENERIC »  
MODIFIED GRAVITY  
MODELS

Hey, wait a minute  
we just started!  
Work by  
Wangzheng Zang  
and collaborator  
(Y.Dubois., etc)  
@IAP

RELATIVISTIC  
RAY-TRACING

**This  
Talk**  
(work by  
**M-A. Breton,**  
**M. Corioni,**  
**F. Castillo, etc.)**

2 POINTS STATISTICS  
(3x2pts)

HIGHER ORDER  
STATISTICS

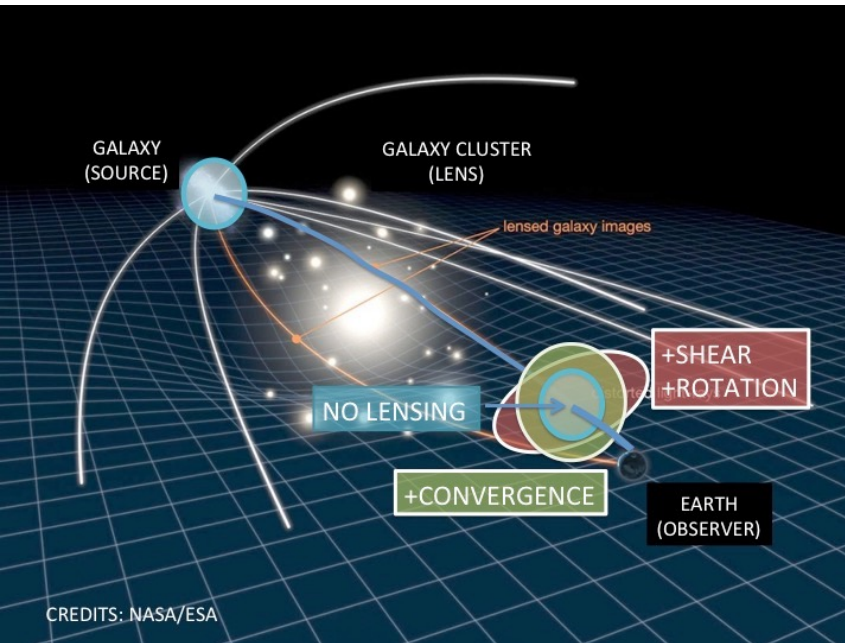
Talk  
work @OAS  
by Fabien  
Castillo  
More in the  
future @  
LUX

EMULATION  
PREDICTIONS

Talk of  
Last year  
About  
e-MANTIS  
Work by  
I.Saez-  
Casares

# Usual approach: compute WL maps or RSD catalogs

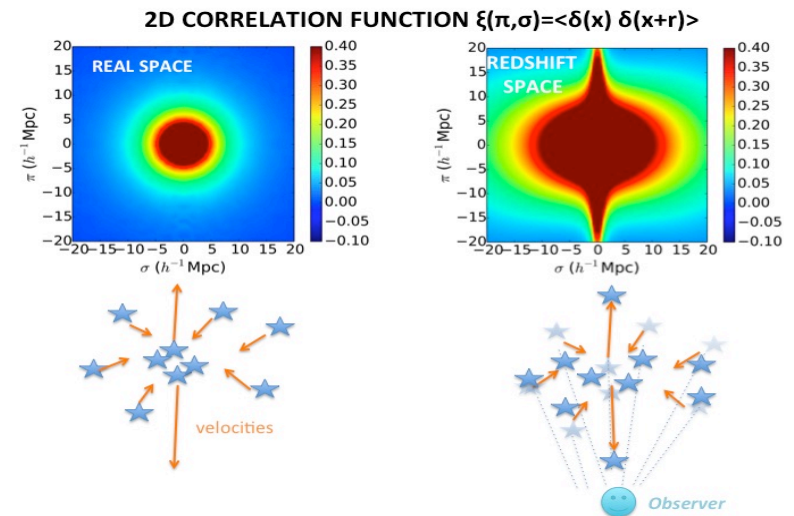
## LENSING



Many approximations-> Example of approximations: no-RSD, flat sky, Born, multiple-lens, replications

OR

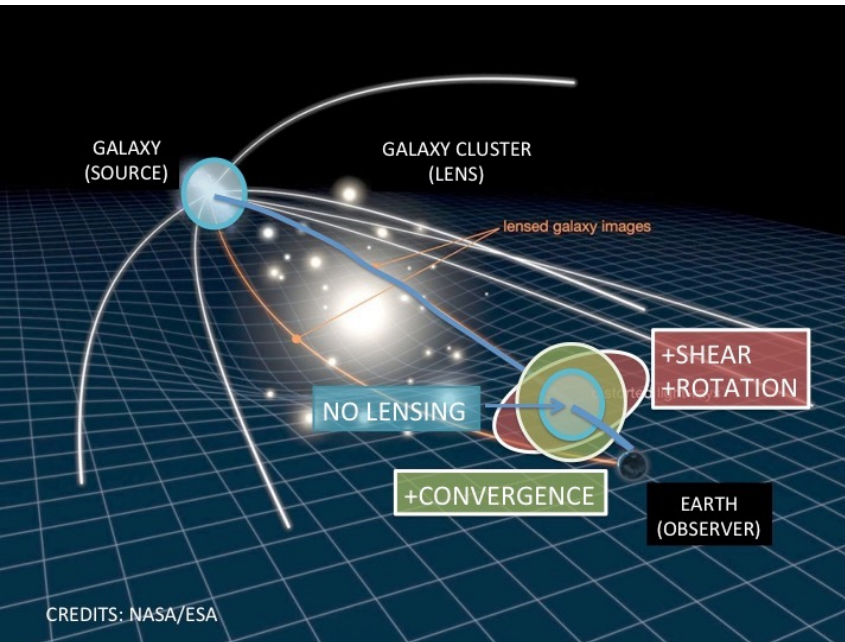
## Redshift-Space Distortions (RSD)



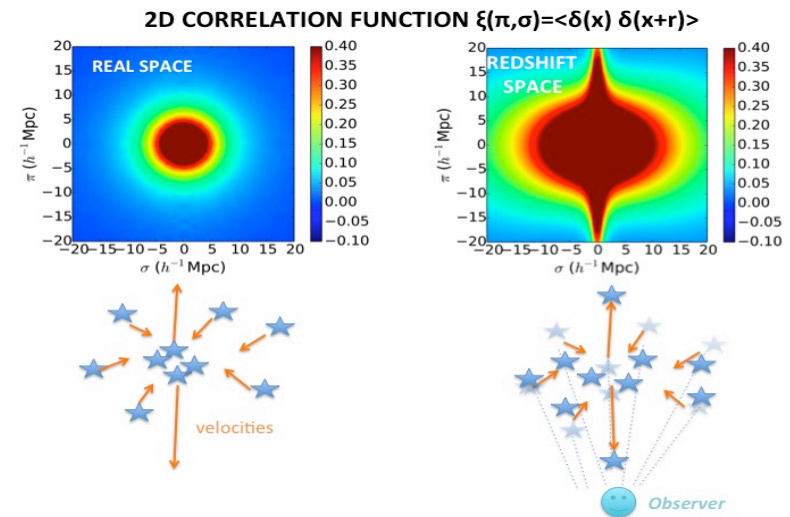
Many approximations-> Example of approximations: no-lensing, distant observer, no gravitational redshift (i.e. Doppler only), no light-cone effect

# Relativistic approach: compute what is really observed following (weak-field) GR

## LENSING



## Redshift-Space Distortions (RSD)



AND

**AND OTHERS** (gravitational redshift, ISW effect, transverse Doppler, etc)

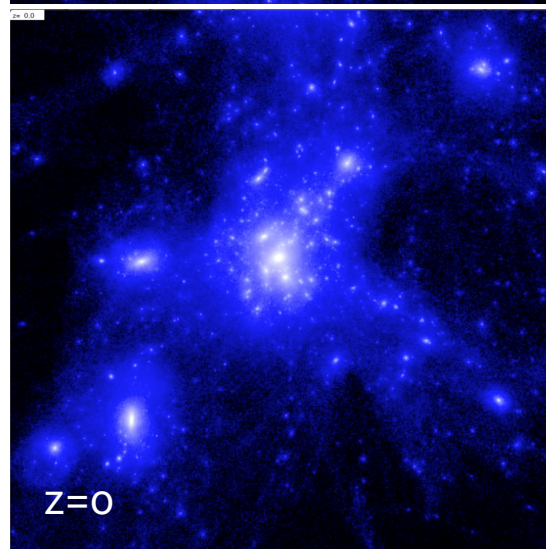
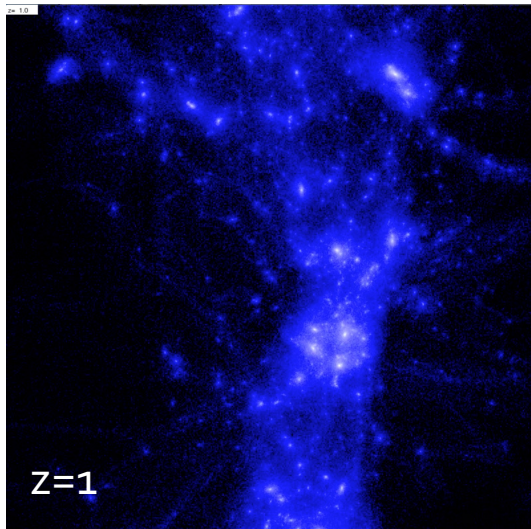
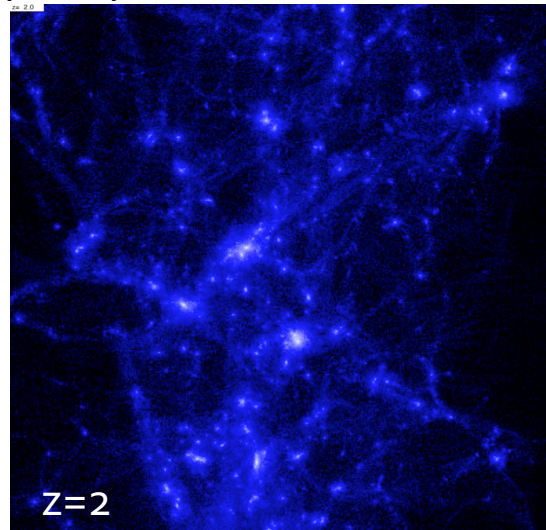
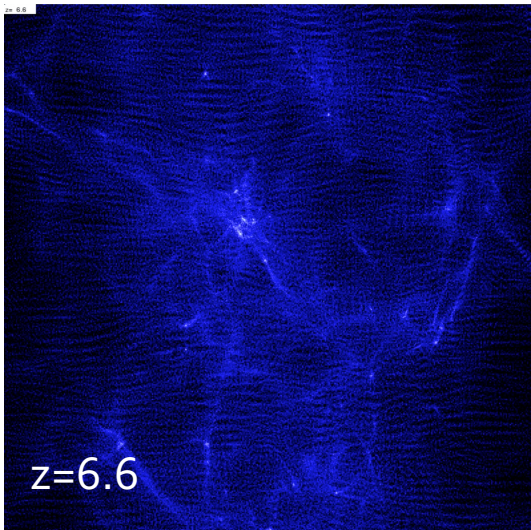
- Relativistic approach at large scales: Yoo+ 2010; Bonvin&Durrer 2011; Yoo 2011; Lewis&Challinor 2011
- Use **similar formalism as for CMB** (i.e. weak field GR) but applied to galaxies  
-> LIMITATION OF ORIGINAL WORKS: **LINEAR REGIME**
- Relativistic approach at cluster scale and around: Kaiser2013, Zhao2013, Croft2013, Cai+2017  
-> LIMITATION: How to connect with linear predictions ?

=> GR effects WITH SIM IS A HOT TOPIC: Killedar12, Reverdy14, Adamek16, Giblin17, Borzyszkowski17, Breton19, Adamek19, Leporizo, Guandalin21, Leporiz1, Raser22, ... 5

## HOW TO PROCEED?

### STEP 0: RUN AN N-BODY SIMULATIONS (e.g. RayGal)

- Goal: Build a (virtual) « real » universe by running N-Body sims with RAMSES
- Specs:  $4096^3$  particles,  $(2.6 \text{ Gpc}/h)^3$ ,  $\Lambda\text{CDM}$  &  $w\text{CDM}$  ( $w=-1.2$ )
- #halos: >10 millions of halos from Milky-Way size to cluster size



*Illustrative  
example of  
the formation  
of one large  
halo in a  
simulation*

# WHAT IS RAMSES ?

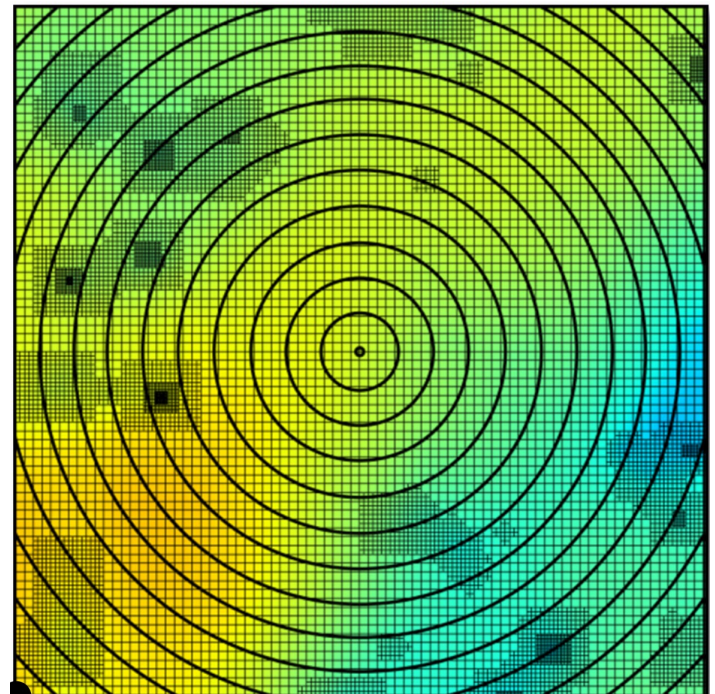
# WHAT IS RAMSES ?

JUST KIDDING.... GOOD NEWS I CAN SKIP THIS SLIDE...

# HOW TO PROCEED?

## Step1: BACKUP A PARTICLES AND A GRAVITY LIGHTCONE

- What: particles lightcone (see Fosalba et al.2008, Teyssier et al. 2009) + gravity lightcone
- Where: At light-travel distance from the observer (center of the box)
- Remark: also backup in the vicinity of the null-FLRW lightcone (called « thick » light-cone)
- Which quantities: Cells: potential (i.e. metric), gradient of the potential (i.e. gravitational field), time derivative of the potential + particles
- Type of light-cone: wide (fullsky,  $z_{\text{max}}=0.5$ ), deep ( $2500 \text{ deg}^2, z_{\text{max}}=2$ ), very deep ( $400 \text{ deg}^2, z_{\text{max}}=10$ )



# STEP 2 : DIRECT INTEGRATION OF BILLION WEAK-FIELD GEODESICS EQUATIONS IN PERTURBED FLRW WITHIN AMR GRID

- Geodesic equations:

$$\frac{d^2 x^\alpha}{d\lambda^2} = -\Gamma_{\beta\gamma}^\alpha \frac{dx^\beta}{d\lambda} \frac{dx^\gamma}{d\lambda}$$

- Redshift definition:

$$1 + z = \frac{\nu_s}{\nu_o} = \frac{(g_{\mu\nu} k^\mu k^\nu)_s}{(g_{\mu\nu} k^\mu k^\nu)_o}$$

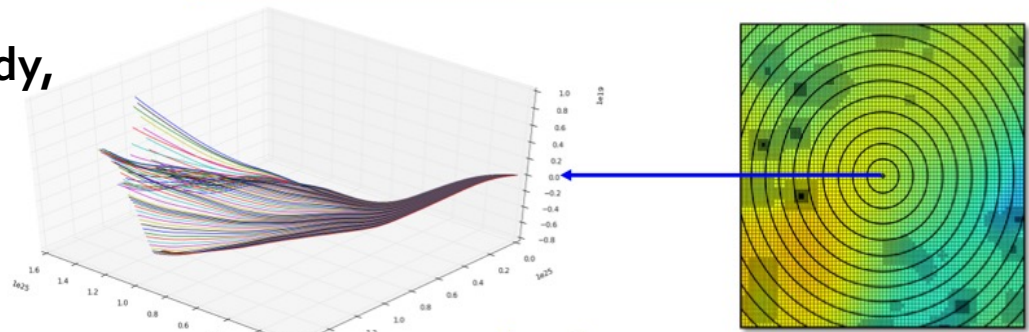
- **MAGRATHEA library (Reverdy 2014):**  
optimized/light AMR (MPI+p-threads)

- **MAGRATHEA-PATHFINDER: ray-tracing, WL, RSD (Breton&Reverdy, 2021)**

- Self-consistent calculation of WL AND RSD AND other relativistic effects

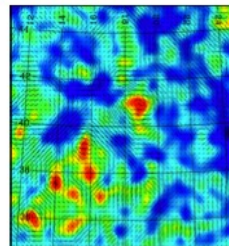
- Little number of controled assumptions: weak-field GR + neglect horizon-scale GR effects on DM dynamics (Chisari & Zaldarriaga, 2011, Adamek et al. 2016)

## 3D backward raytracing

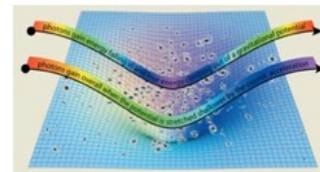


For free...

**Weak lensing**  
(convergence & shear)



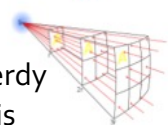
**Integrated Sachs-Wolfe**



**Luminosity distance**  
**Angular distance**  
**Redshift distortions**  
**Time delays**

...

V. Reverdy  
thesis



# STEP 3: MAGRATHEA-PATHFINDER'S ITERATIVE GEODESICS FINDER

*Breton et al, 2019, 2022*

## Find null geodesics

*Find the connection between  
Observer  $O$  and Source  $S$*

*Using Newton's method :*

$$x = (x_1, \dots, x_n)$$

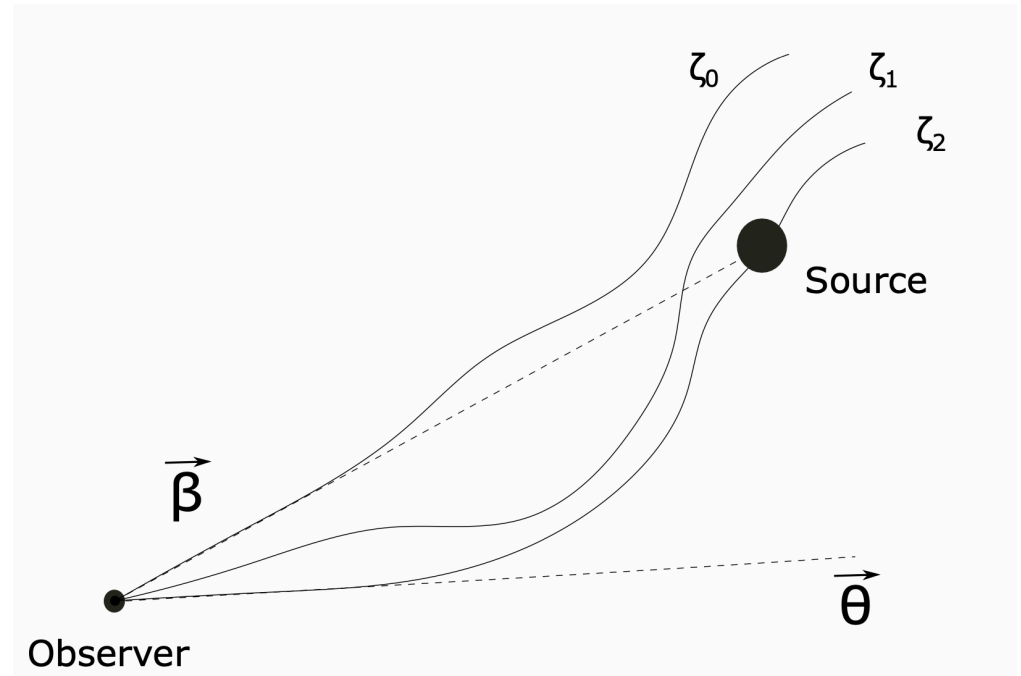
$$x_{k+1} = x_k - F(x_k)/F'(x_k)$$

## Output

*« NEW » : Catalogs of sources  
taking into account weak  
lensing effects and redshift  
space distortions*

*In the catalogs :*

$\vec{\beta}$ ,  $\vec{\theta}$ ,  $\bar{z}$ ,  $z$ , errors,  $A_{ij}$

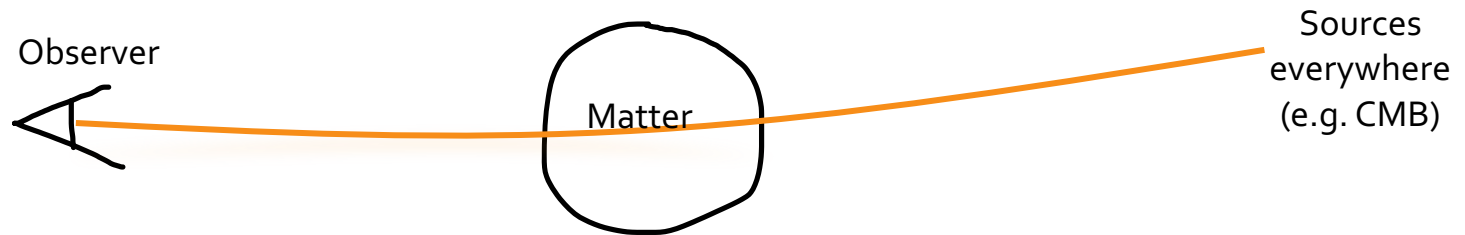


- Weak-lensing using the ray-bundle approach
- Launch a beam of photons and directly compute its distortion (i.e. distortion matrix)
- Account for finite beam effect (i.e. the size of a galaxy is not zero as in the usual WL formalism)

# IMPORTANT TAKE HOME MESSAGE

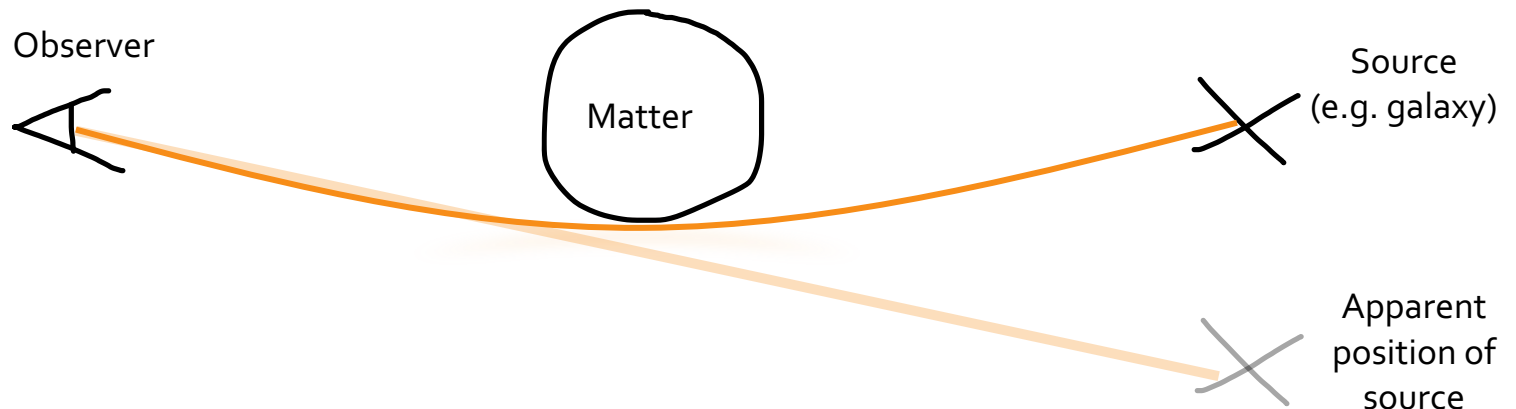
## SOURCE AVERAGING VERY DIFFERENT FROM ANGULAR AVERAGING

### ANGULAR AVERAGING:



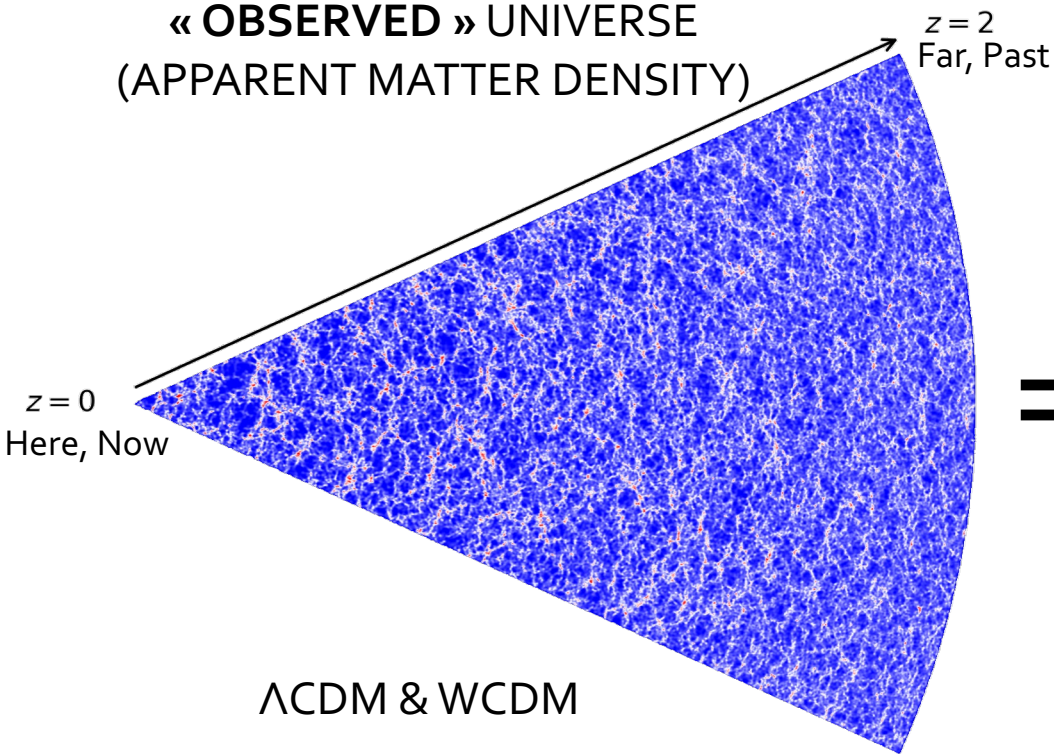
Usual approach: Born, post-Born, multiple-lens => minor correction

### SOURCE AVERAGING:



In Magrathea, geodesics finders => light crosses exactly the source and the observer  
**strong effects (magnification bias etc), not only a post-born correction**

SIMULATION  
OF A SLICE OF THE  
« **OBSERVED** » UNIVERSE  
(APPARENT MATTER DENSITY)

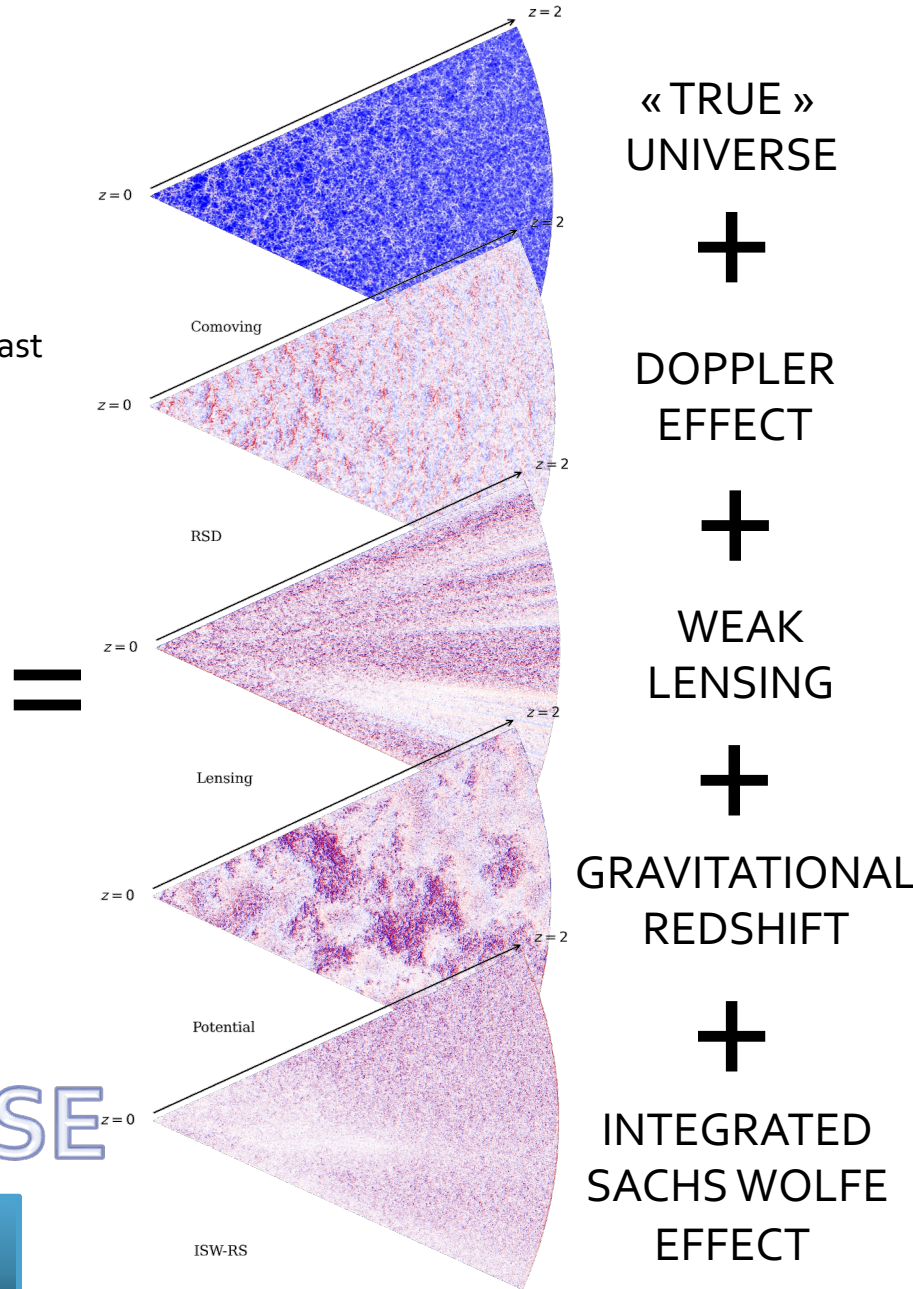


# THE RAYGAL UNIVERSE

70 billion particles  $\Rightarrow$  cosmic structure formation  
1 billion photons  $\Rightarrow$  general relativistic effects

OPEN DATA: <https://cosmo.obspm.fr/public-datasets/>

(or type « **RayGal data** » on any search engine)



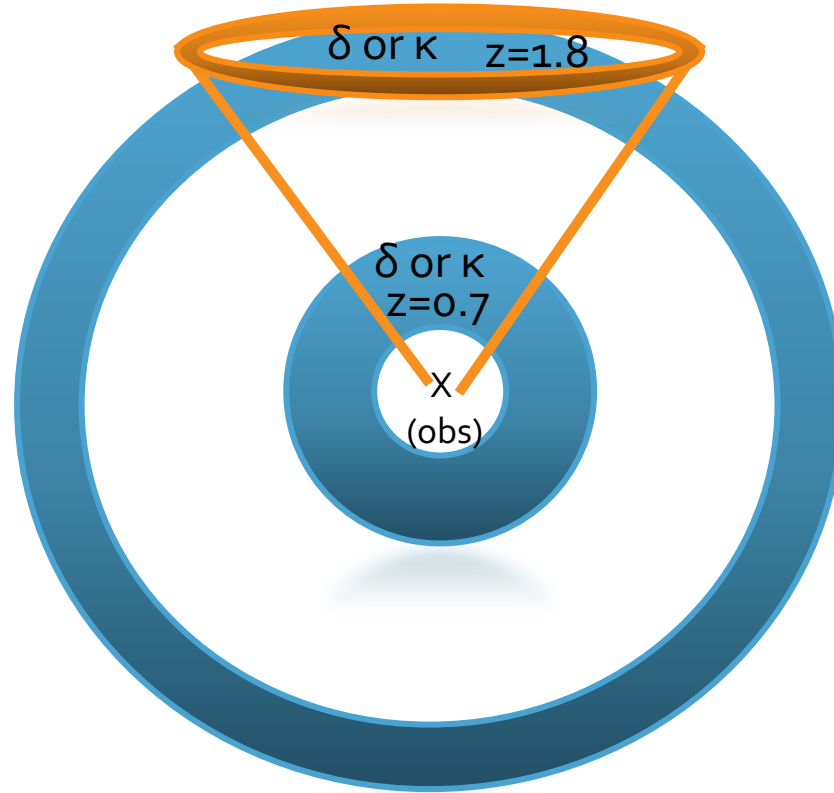
# **Example of application: relativistic 3x2pts correlations**

**(I will skip a natural application which is**

**relativistic redshift-space distortions**

**see Breton+19 for this)**

# GEOMETRY AND QUANTITIES



- Quantities: Observed matter overdensity  $\delta$  and gravitational weak-lensing convergence  $\kappa$

- Statistics:  $C_l$   
 $\langle \delta \delta \rangle$  : clustering  
 $\langle \kappa \kappa \rangle$  : weak-lensing  
 $\langle \delta \kappa \rangle$  : galaxy-galaxy lensing

- Geometry: centered on observer, 2500 deg<sup>2</sup> light-cone : shells at  $z=0.7 \pm 0.2$  and  $z=1.8 \pm 0.1$

- DEFINITION:

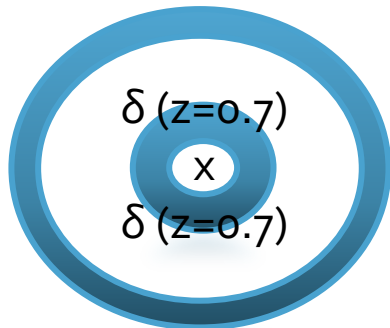
**NON-TRIVIAL RELATIVISTIC EFFECTS= DEVIATION FROM COMOVING MATTER OVERDENSITY AND BORN CONVERGENCE** (ie. mostly magnification bias MB and RSD)

- THEORY (for comparison): CLASS

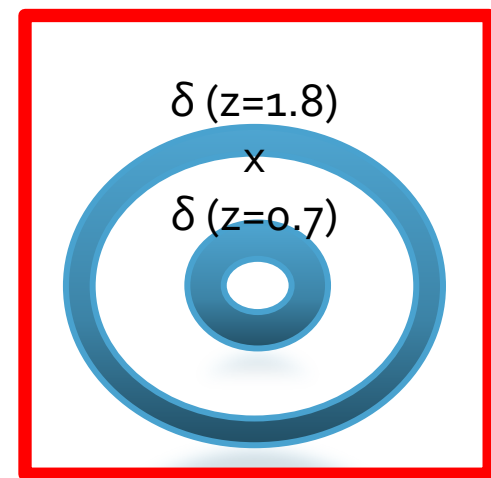
-> Linear with all relativistic effects

+ Non-linear prescription= halofit, linear mapping, RSD no Finger of God, Born lensing, etc.

## Clustering (density x density)

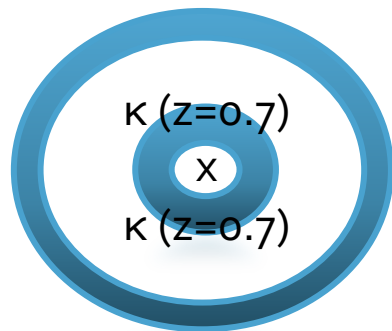


$$\delta(z=1.8) \times \delta(z=1.8)$$

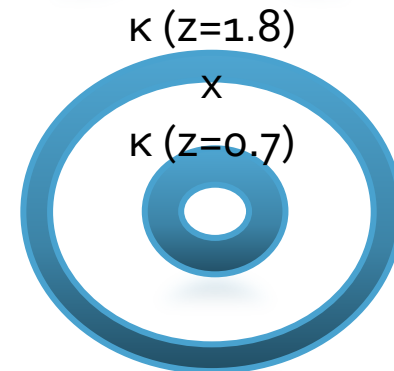


Purely  
relativistic  
often  
omitted

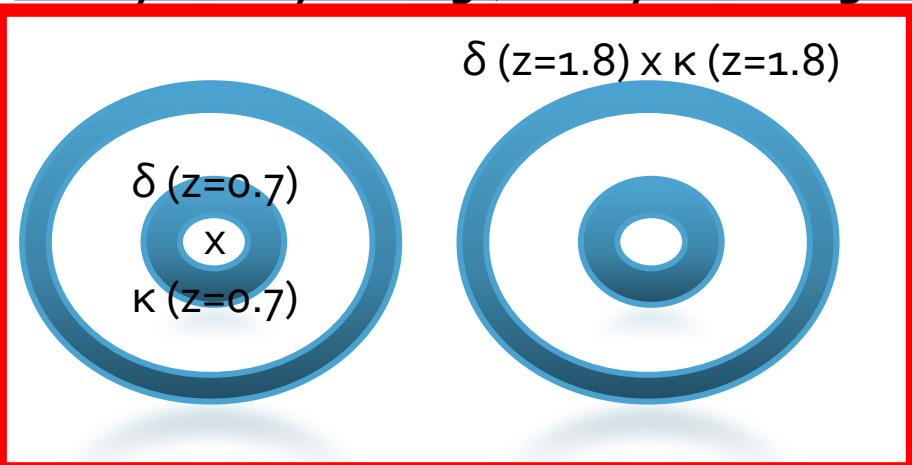
## Weak-lensing (convergence x convergence)



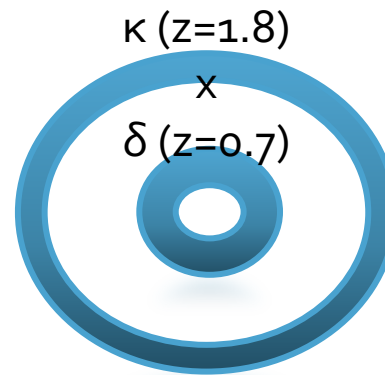
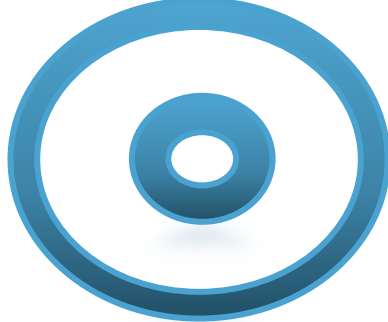
$$\kappa(z=1.8) \times \kappa(z=1.8)$$



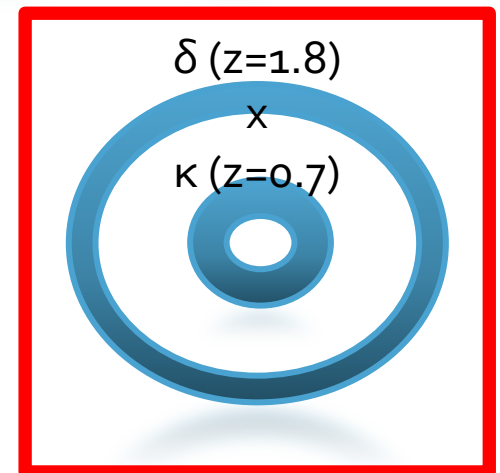
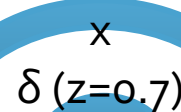
## Galaxy-Galaxy lensing (density x convergence)



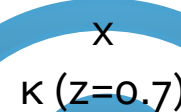
$$\delta(z=1.8) \times \kappa(z=1.8)$$



$$\kappa(z=1.8) \times \delta(z=0.7)$$

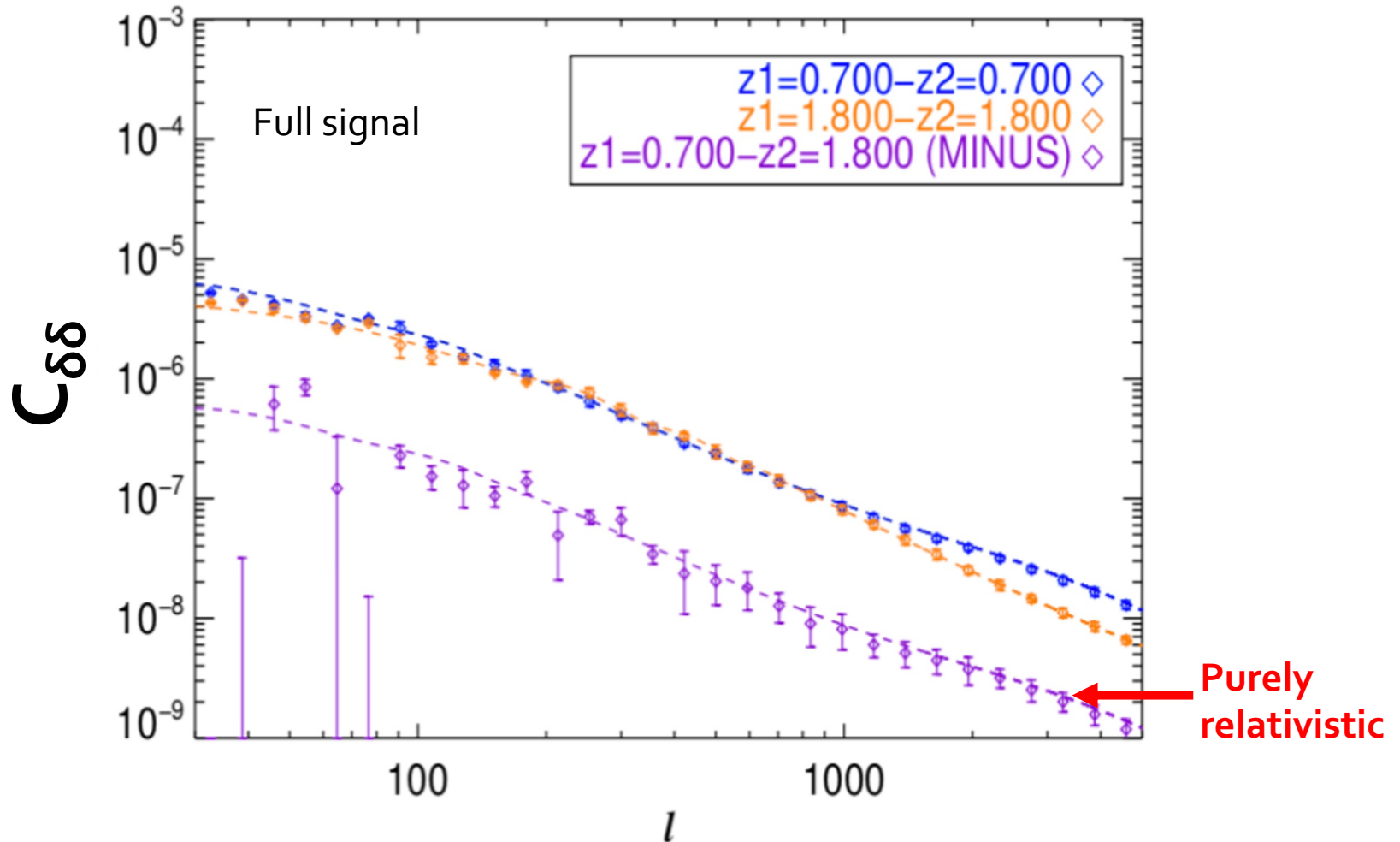


$$\delta(z=1.8) \times \kappa(z=0.7)$$

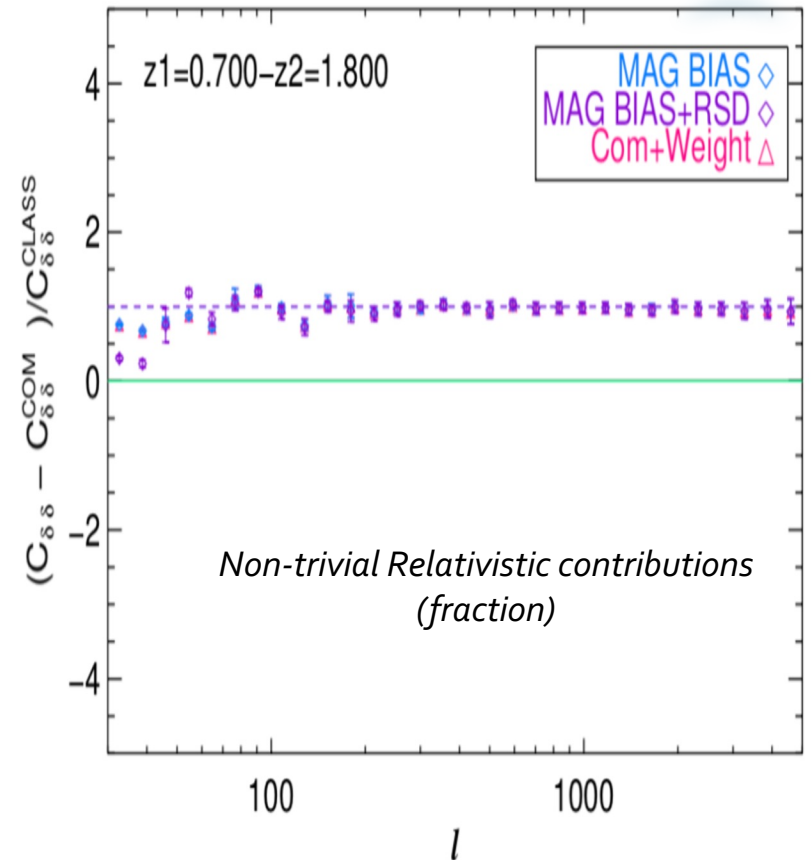
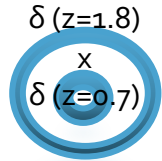
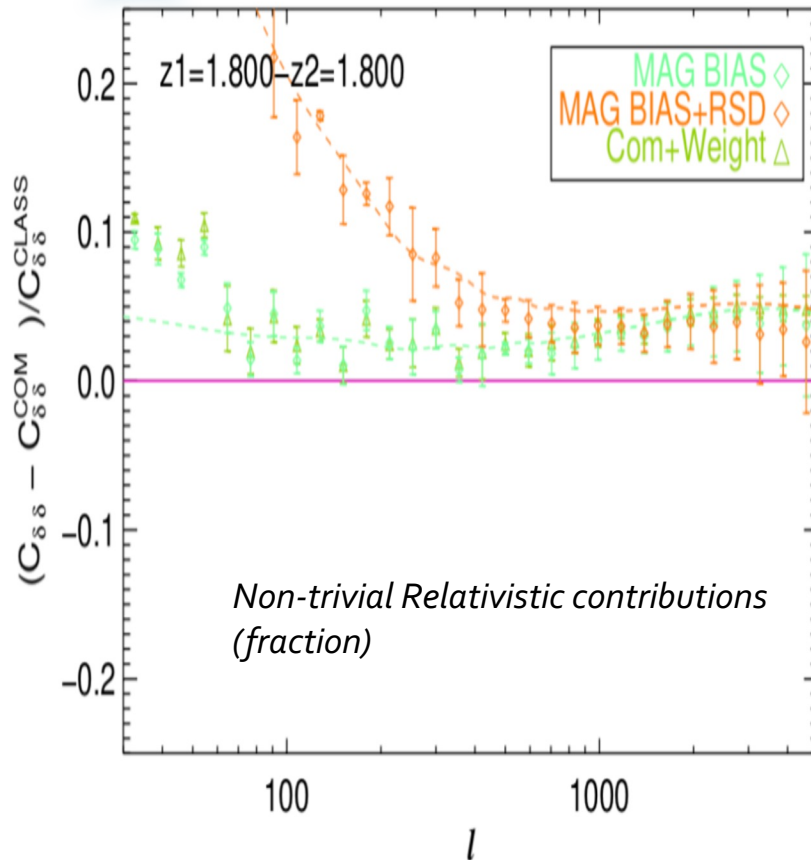
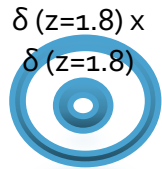


# Matter angular (cross-)power spectra with relativistic contributions

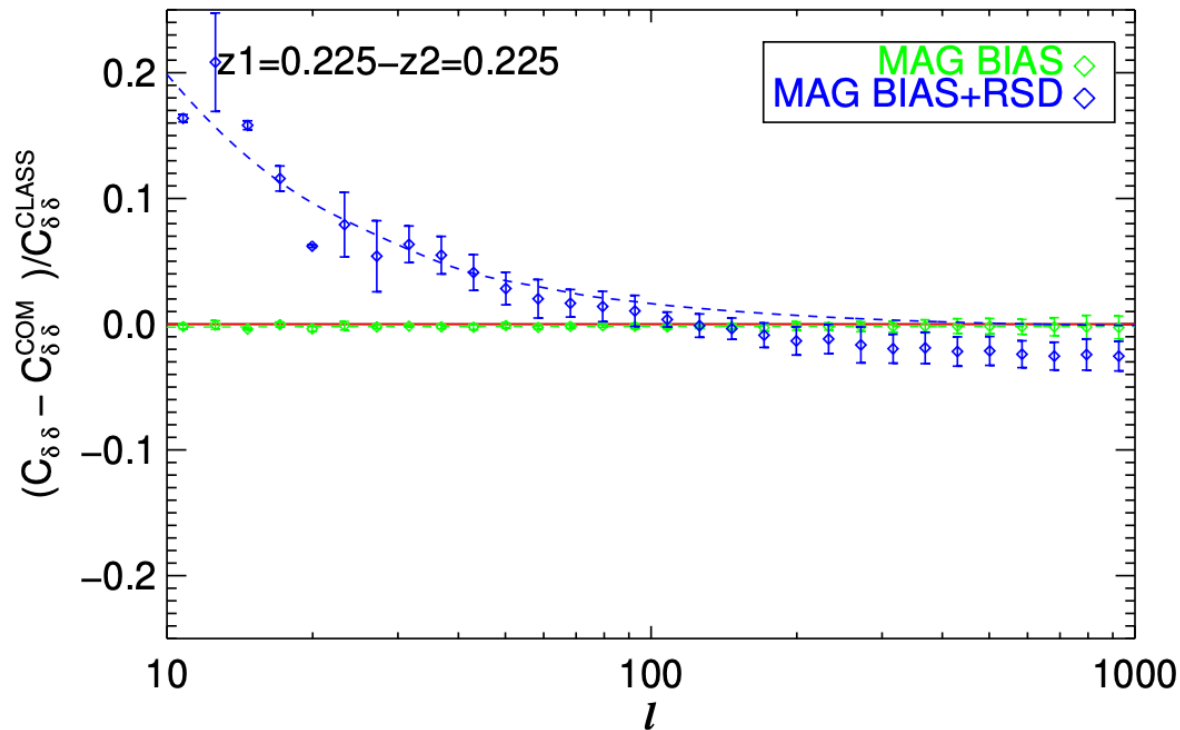
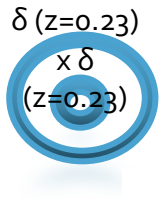
Rasera et al. 2022



- **Good agreement with Class** (dashed lines)
- 3D matter  $P(k)$  calibrated on RayGal (otherwise halofit errors induce  $\sim 5\%$  errors)

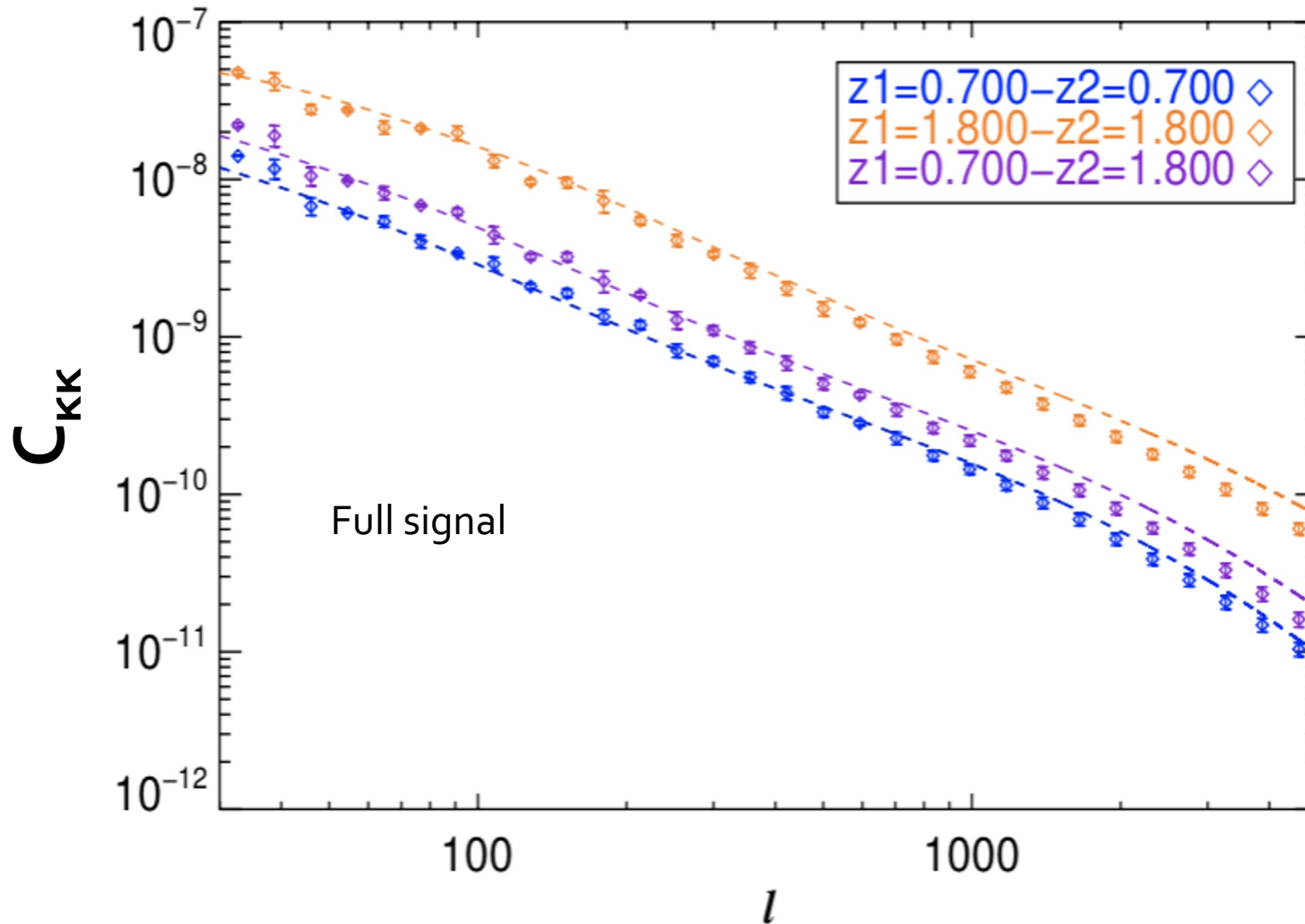


- RSD effect at large scale
- Magnification Bias (MB)  $\Rightarrow \delta_{\text{obs}} \approx \delta_{\text{com}} - 2\kappa_{\text{Born}}$  (for flux-limited survey  $-2\kappa \rightarrow (5s - 2)\kappa$ )
- MB effect at every scale (+ dominate for distance shell)



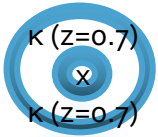
- Remark: Same autocorrelation but at smaller redshift  $z=0.225$
- Agreement at large scale but **Class doesn't capture Fingers-of-god effect at small scales**

Convergence angular (cross-)power spectra  
spectra with relativistic contributions

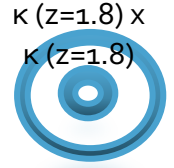
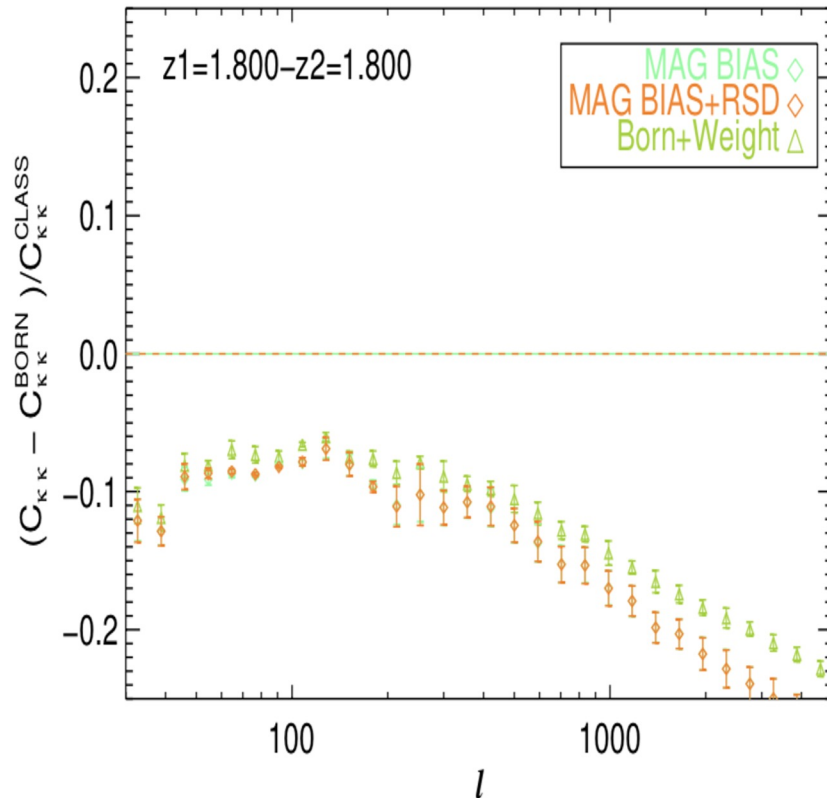


- Reasonable agreement with class at large scales

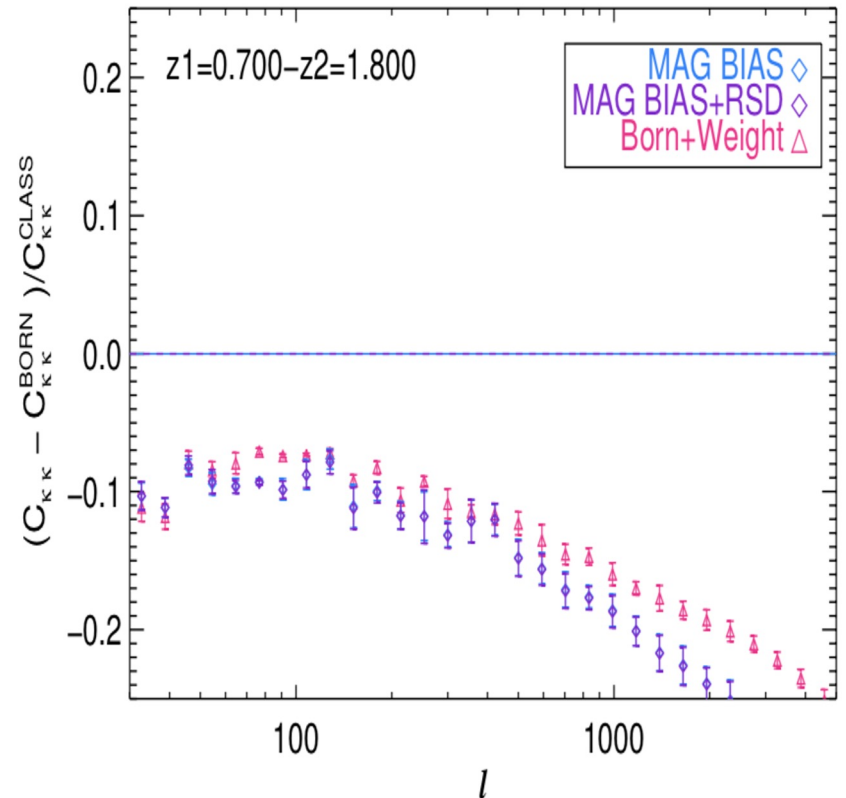
# Convergence angular (cross-)power spectra: magnification bias and RSD effect



Non-trivial Relativistic contributions (fraction)

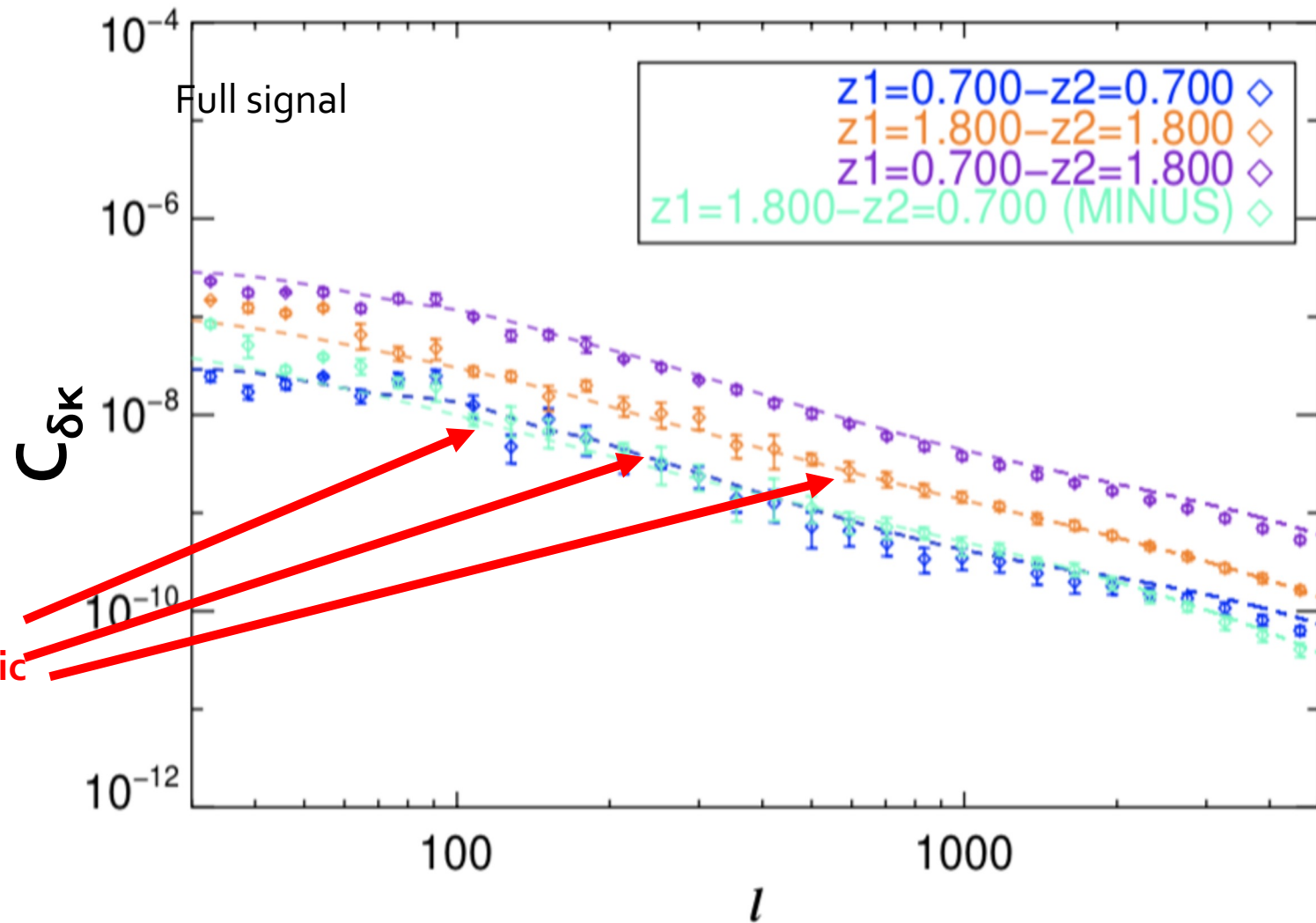


Non-trivial Relativistic contributions (fraction)



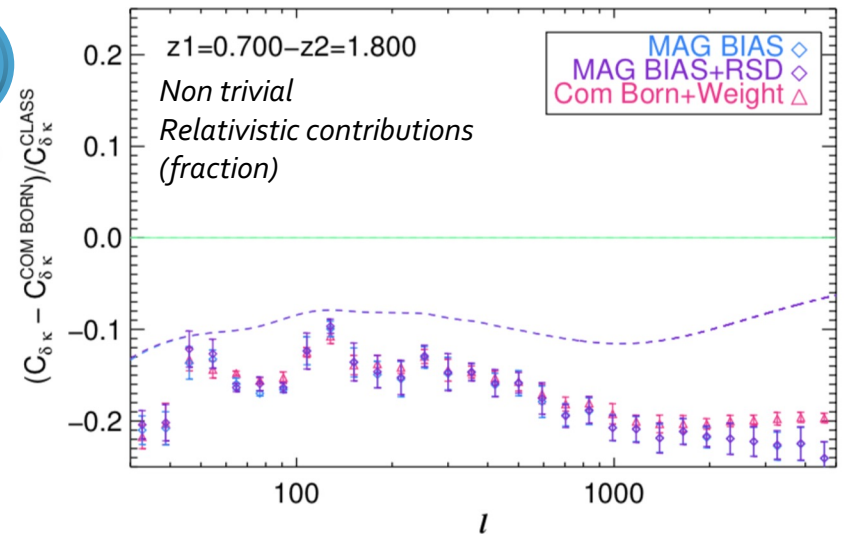
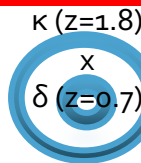
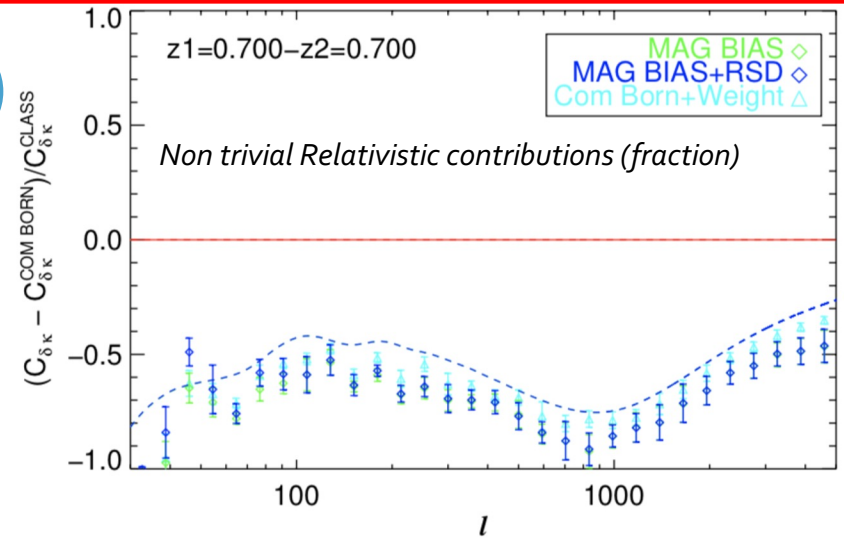
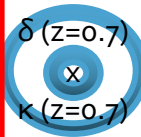
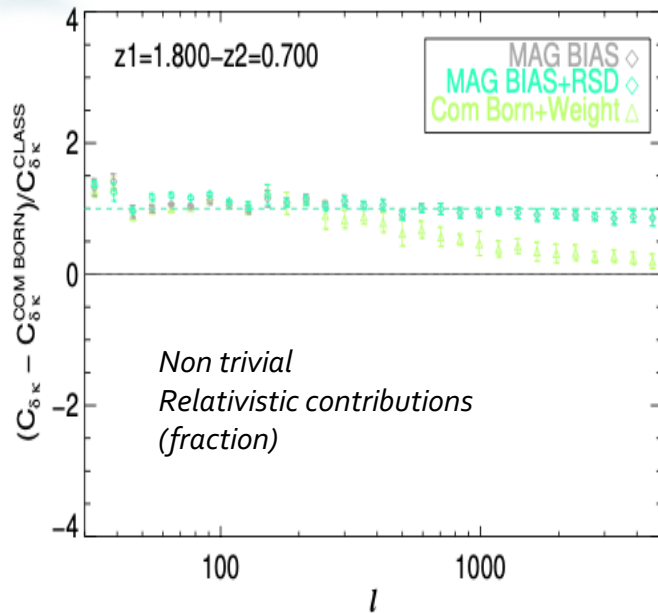
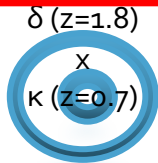
- Cannot compute the effect of magnification bias on the convergence with Class (as it is related to the bispectrum)
- $\kappa_{\text{obs}} \approx \kappa_{\text{Born}} (1 - 2 \kappa_{\text{Born}})$
- MB effect on convergence Cl means that **shear and convergence power spectra differ!**

# Density-convergence (cross-)power spectra with relativistic contributions



- Good agreement with Class

# Density-convergence (cross-)power spectra: magnification bias and RSD effect

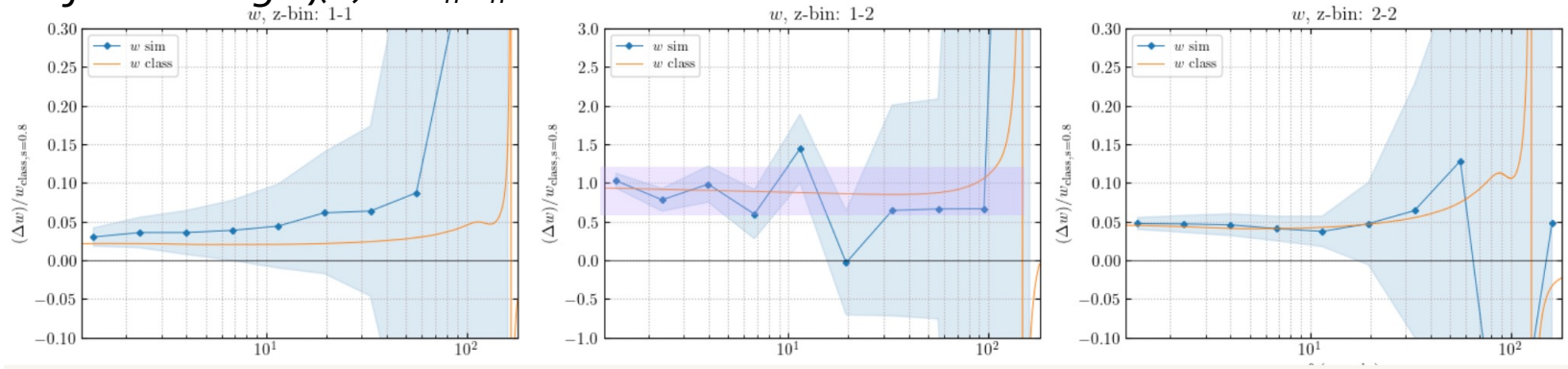


- MB  $\Rightarrow \delta_{\text{obs}} \approx \delta_{\text{com}} - 2\kappa_{\text{Born}}$  &  $\kappa_{\text{obs}} \approx \kappa_{\text{Born}} (1 - 2\kappa_{\text{Born}})$ .
- MB effect in Class is included but only for the density not the convergence  $\Rightarrow$  deviations
- Interesting **non-trivial configurations** : including some with the convergence at lower <sup>23</sup> or equal redshift than the density shell  $\Rightarrow$  the cosmological signal is not negligible

**ONGOING WORK (M.Corioni, F.Castillo)**  
**Going closer to observations and exploring more models...**

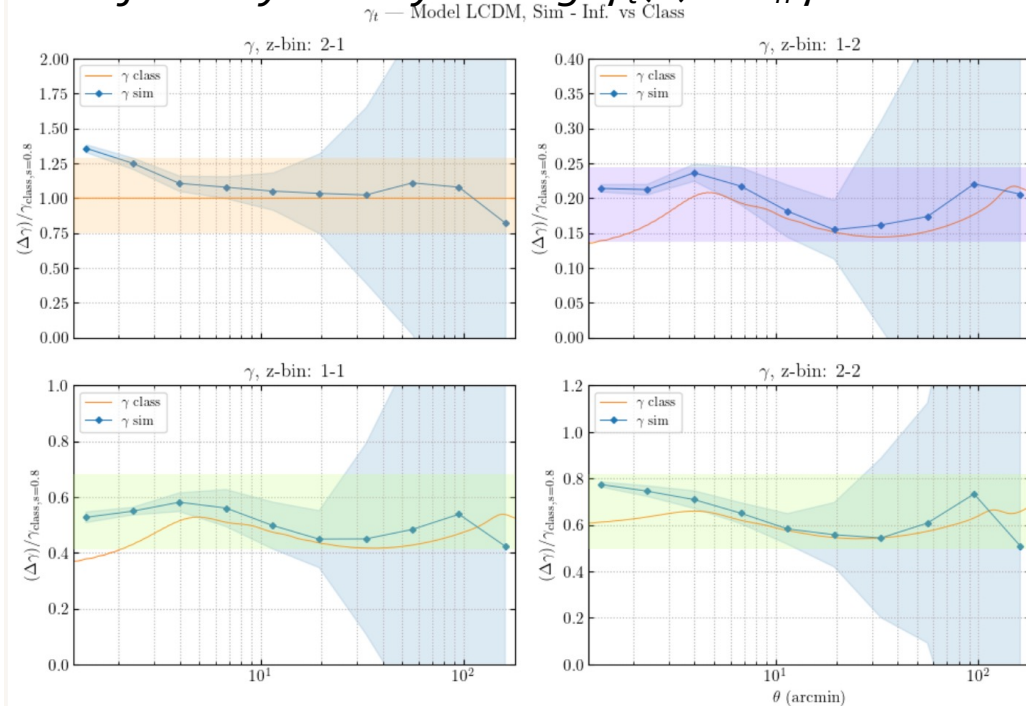
- Observed matter overdensity  $\delta$  -> Halo overdensity (in the future galaxies)
- Gravitational weak-lensing convergence  $\kappa$  -> shear  $\gamma$
- Volume limited survey -> Flux-limited survey
- ProGraceRay simulation suite to explore wo-wa-CDM space (ongoing)
- Focusing on small scale in configuration space: angular power Spectrum -> angular correlation function

# MB of Clustering: $\xi(\vartheta) = \langle \delta_h \delta_h \rangle$ Magnification bias (MB) of 3x2pts correlation functions (preliminary)

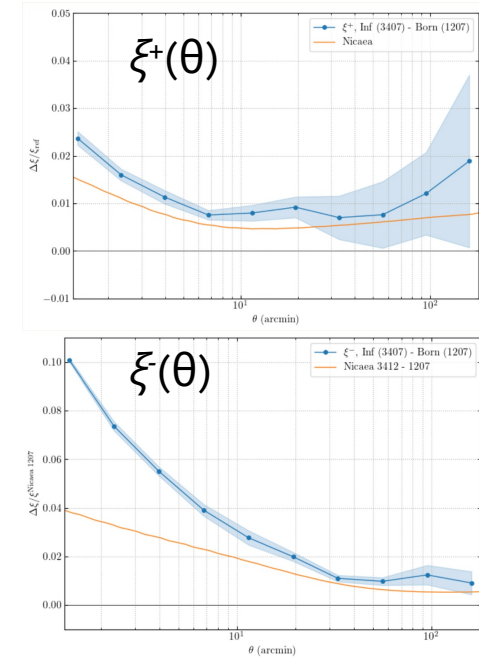


**Crédits:**  
**M. Corioni**

## MB of Galaxy-Galaxy lensing: $\gamma_t(\vartheta) = \langle \delta_h \gamma \rangle$



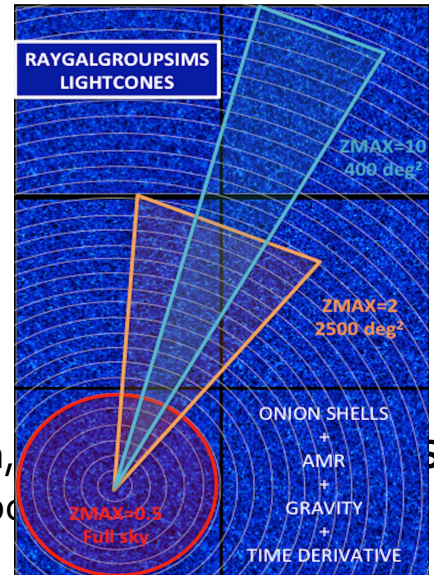
## MB of shear correlation $\langle \gamma \gamma \rangle$



- Similar results... but MB effects on the shear is smaller since  $\langle \gamma \gamma \rangle < \langle \kappa \kappa \rangle$
- MB predictions on shear correlation function by Nicaea  $\rightarrow$  large scales only
- Study on going... Cosmology dependence... etc. (F.Castillo)

# CONCLUSION

- **Goal:** Understand the **connection from the “real universe” to the “apparent universe”** to find new probes of DE=> need to model **all weak-field relativistic effect** (i.e. like for CMB but in non-linear regime)
- New **PUBLIC DATA**
  - Don't hesitate to download the **RAYGALGROUPSIMS** (or in short **RayGal**) relativistic halo catalogues and maps to make your own test (traditional snapshot data are also available)
  - Very simple files with angular position, redshift and distortion matrix
  - Magrathea geodesics-finder => SOURCE AVERAGING available
- **Relativistic effects and weak-lensing (3x2pts):**
  - good agreement with CLASS at quasi-linear scales
  - **subtle effects in NL regime** (Finger-of-gods effect in angular correlation, on the convergence power spectra, **non-trivial configuration in GGL**)=> p
- **Relativistic effects in RSD => not shown here see Breton+19**
- **Very general approach, many extensions:**
  - Many Other possible applications (theory/simulation/observation) : doppler lensing, ISW, fluctuations of cosmic distances, cluster studies (WL, RSD, gravitational redshift), etc...
- **Ongoing work: towards more realistic catalogs in modified gravity cosmologies...**



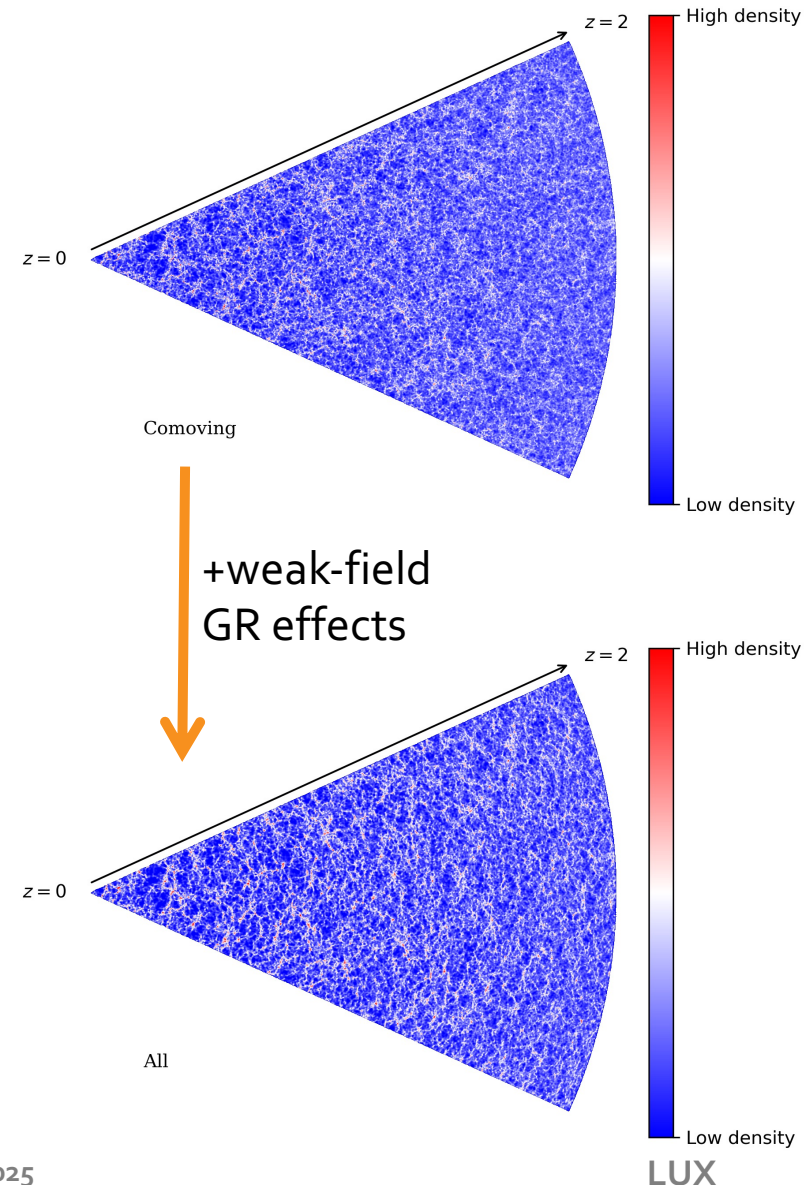
# BACKUP SLIDES

# RayGal simulation suite with General Relativistic Ray-Tracing

Breton et al. 2019;  
Rasera et al. 2022

*Weak-field GR approach from linear to non-linear scales...*

- Large and well resolved HPC N-body simulations ( $4096^3$  part.  $L=2.625$  Gpc/h)
- Standard cosmology ( $w=-1$ ) + **alternative dark energy model** ( $w=-1.2$ )
- **Ray-tracing** including all **general relativistic effects** in the weak field regime at high-resolution
- **Billion light-rays** launched
- For the first time, **identification of light rays going exactly from the source to the observer.**
- Unique **halos catalogues** including **beyond state-of-the-art weak-lensing and redshift space distortions** (Doppler effect, gravitational redshift, weak-lensing, ISW).



# Very generic, built from 1st principles=> many applications

*A laboratory to test relativistic effects...*

$\langle dL \rangle$  &  $\langle dA \rangle$ : Bias of distance-redshift relation (Breton&Fleury, 2021)  
 $\langle \delta_{\text{gal}} \delta_{\text{gal}} P_{0,2,4} \rangle$ : Magnification bias in RSD (Breton et al, 2022)  
 $\langle \delta_{\text{halo}} \delta_{\text{halo}} P_1 \rangle$ : Dipole in RSD (Breton et al. 2019 Taruya et al. 2020 Saga et al. 2020, 2021)  
 $\langle \delta \delta \rangle, \langle \delta \kappa \rangle, \langle \kappa \kappa \rangle$ : WL Convergence x matter overdensity (Rasera et al. 2022)

Possible examples:

$\langle \delta_{\text{gal}} \delta_{\text{gal}} \rangle, \langle \delta_{\text{gal}} \gamma \rangle, \langle \gamma \gamma \rangle$ : Relativistic effects on 3x2pts => **Doable**

$\langle \gamma \gamma \rangle$ : Finite beam lensing => **Doable**

$\langle w w \rangle$  => **Doable (to be checked)**

$\langle z_{\text{grav}} \rangle$ : Gravitational redshift in cluster/voids (also with TD, WA&LC effect)= **Doable**

$\langle \delta_{\text{gal}} \text{ISW} \rangle, \langle \gamma \text{ISW} \rangle$  => **Doable**

$\langle \delta_{\text{gal}} \delta_{\text{gal}} \delta_{\text{gal}} \rangle, \langle \gamma \gamma \gamma \rangle, \langle \delta_{\text{gal}} \delta_{\text{gal}} \text{ISW} \rangle, \dots$ .....Bispectrum=> **Doable**

Peculiar velocity polyspectra (also with gravitational redshift, etc)=> **Doable**

Etc....

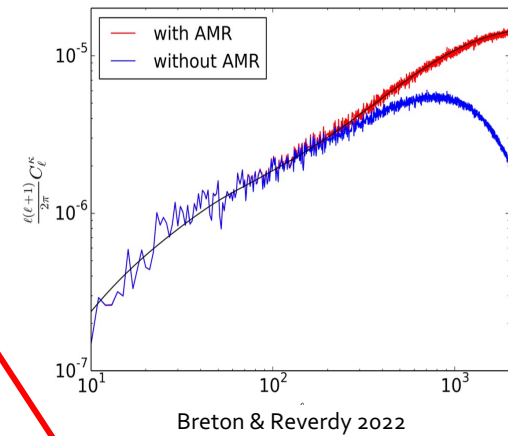
Velocity/potential of the observer => **Implemented in Magrathea, need to be run**

$\langle \text{Flexion } \gamma \rangle, \langle \text{Flexion Flexion} \rangle$  => **Implemented in Magrathea, need to be run**

Gpc clustering (e.g. for PNG)=> **needs to correct w/ analytical models or full GR sims**

*You probably have many other ideas...*

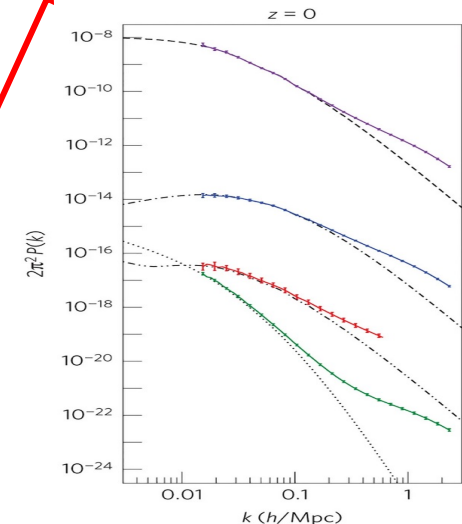
DM DYNAMICS: NEWTONIAN 👎  
 RESOLUTION: ADAPTIVE 👍  
 WEAK-LENSING: GEODESICS INTEGRATION 👍  
 RSD: INCLUDE RELATIVISTIC EFFECTS 👍  
 AVERAGE: ANGULAR OR SOURCE 👍  
 e.g. RayGal sims (this work)



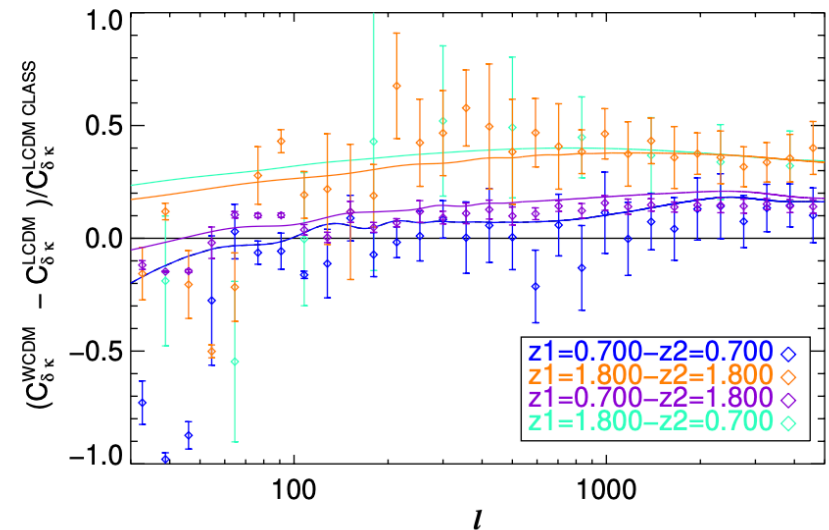
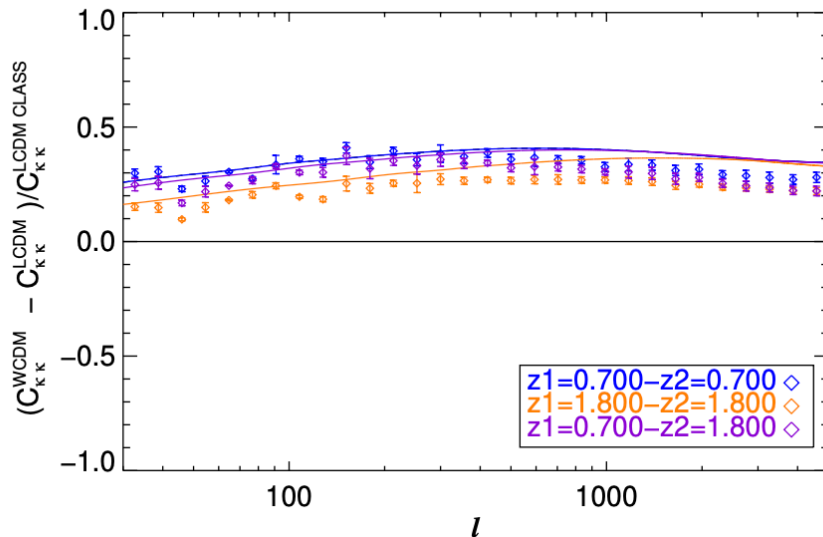
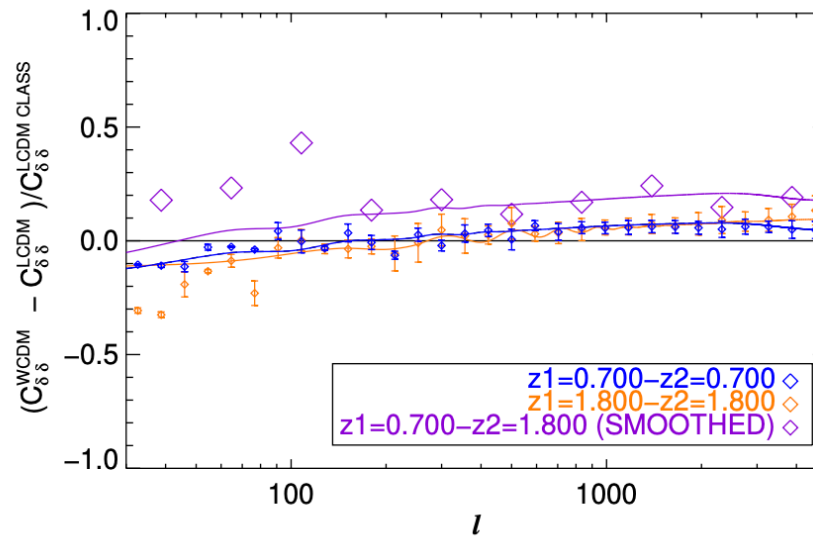
DM DYNAMICS: NEWTONIAN 👎  
 RESOLUTION: ADAPTIVE 👍  
 WEAK-LENSING: BORN 👎  
 RSD: STANDARD 👎  
 AVERAGE: ANGULAR 👎  
 e.g. usual N-body sims

DM DYNAMICS: FULL GR 👍  
 RESOLUTION: ADAPTIVE 👍  
 WL: GEODESICS INTEGRATION 👍  
 RSD: INCLUDE RELATIVISTIC EFFECTS 👍  
 AVERAGE: ANGULAR OR SOURCE 👍  
 e.g. Ultimate futuristic sims

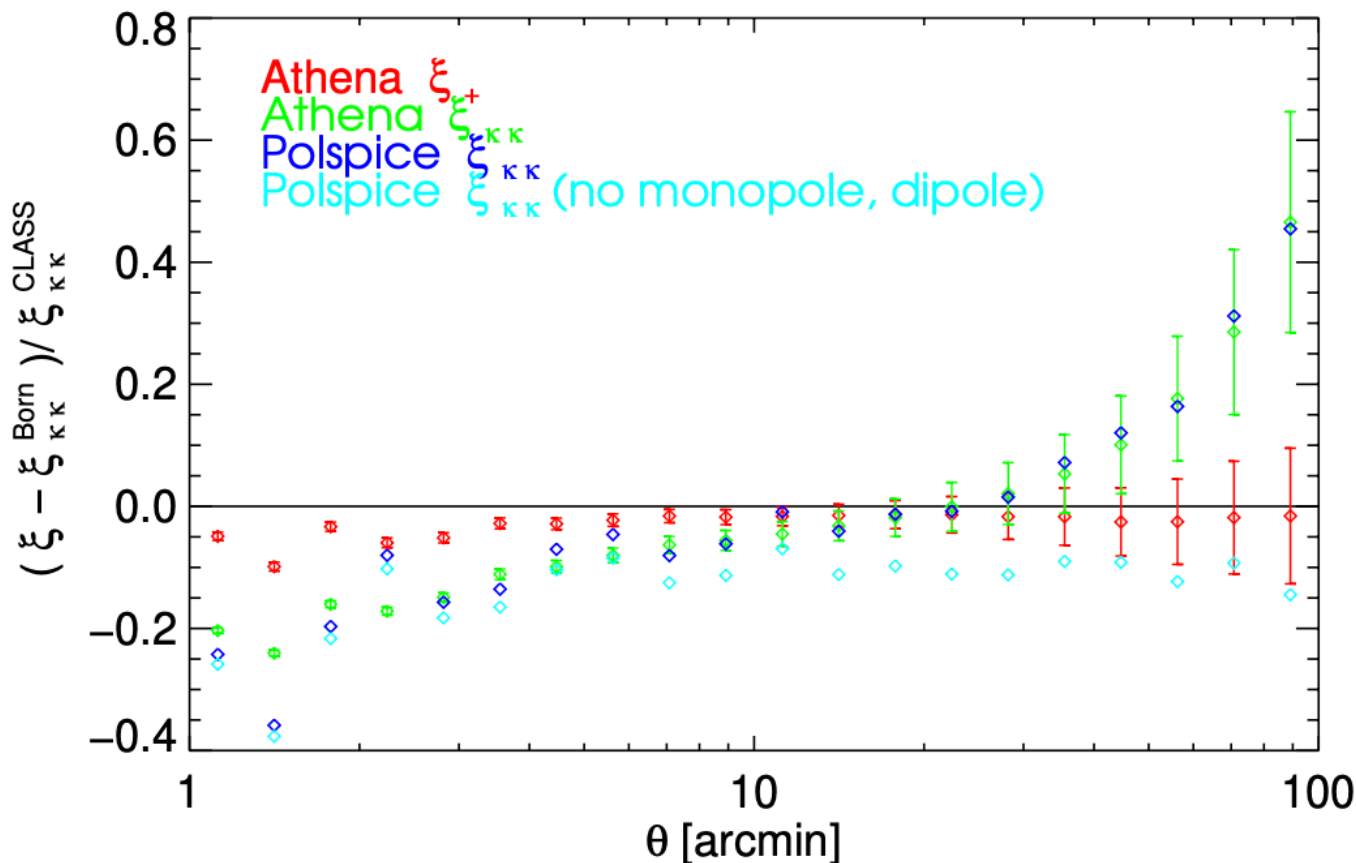
DM DYNAMICS: WEAK-FIELD GR 👍  
 RESOLUTION: FIXED 👎  
 WEAK-LENSING: GEODESICS INTEGRATION 👍  
 RSD: INCLUDE RELATIVISTIC EFFECTS 👍  
 AVERAGE: ANGULAR OR SOURCE 👍  
 e.g. Gevolution sims



# Cosmological dependance (relative difference between $\Lambda$ CDM ( $w=-1.2$ ) & LCDM )



=> **RELATIVISTIC** LENSING-MATTER CLUSTERING IS A POWERFUL COSMOLOGICAL PROBE



**Fig. B.1.** Relative difference between lensing angular two point correlation function on the source catalogue accounting for the dilution bias and the Born convergence angular two point correlation function. In red and green diamonds we show the measurements of cosmic shear and convergence correlation function using ATHENA, and in blue and light blue we show the results using the same methodology as in Sect. 3, keeping and removing the monopole and dipole, respectively.